NAME: ________________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020

S4

ADDITIONAL MATHEMATICS — 4047/01

Paper 1

1 September 2020 Tuesday
2 hours

Candidates answer on the Question Paper
Additional Material: 1 Graph Paper

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiners' Use

Question	Marks	Question	Marks	Table of Penalties	
1		7			
2		8		Units	
3		9		Presentation	
4		10		Accuracy	
5		11		Total:	
6		12			
Parent's Name & Signature:					
Date:				80	

This paper consists of **18** printed pages.   **[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

1 The roots of the quadratic equation $2x^2 - 6x + 1 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Find the value of $\dfrac{\alpha^2 + \beta^2}{\alpha\beta}$. [1]

(iii) Form a quadratic equation with roots $\dfrac{\alpha}{\beta} + 2$ and $\dfrac{\beta}{\alpha} + 2$. [3]

[Turn over

2 The mass, m grams, of a radioactive substance, present at time t days after first being observed, is given by the formula $m = 30\,e^{-0.025t}$.

(i) Find the mass remaining after 30 days. [2]

(ii) Find the number of days required for the mass to drop to half of its initial value. Give your answer correct to the nearest integer. [2]

(iii) State the value m approaches when t becomes large. [1]

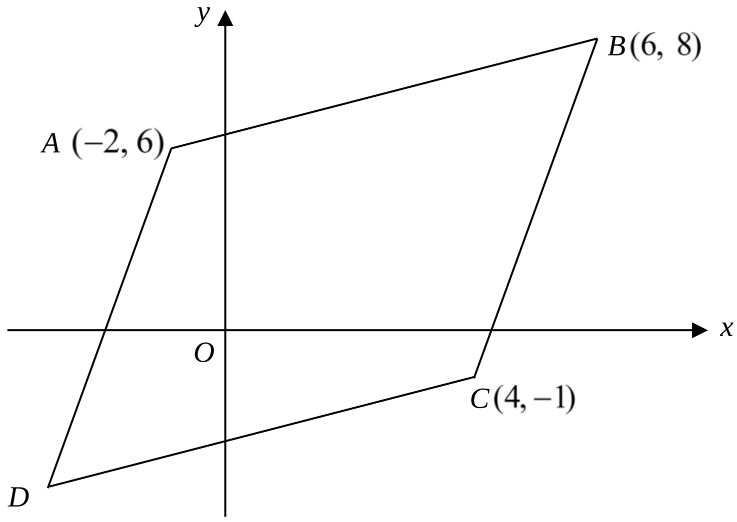
3 **(a)** Find, in ascending powers of x, the first three terms in the expansion of $(2-x)^7$. [2]

Hence, find the value of the constant a for which the coefficient of x^2 in the expansion of $(a-x)(2-x)^7$ is 616. [3]

(b) In the expansion of $\left(x^2-\frac{1}{2x^4}\right)^n$ in descending powers of x, the sixth term is independent of x. Find the value of n and the term independent of x. [4]

[Turn over

4 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a parallelogram *ABCD* in which *A* (– 2, 6), *B* (6, 8) and *C* (4, – 1) are the coordinates of its vertices. Find the

(i) equation of *AD*, [2]

(ii) coordinates of *D*, [2]

(iii) equation of the perpendicular bisector of the line *AD*, [2]

(iv) area of the parallelogram *ABCD*, [2]

(v) acute angle the line *AB* makes with the *y*-axis. [2]

[Turn over

5 **(i)** Show that $\frac{d}{dx}\left(\tan^3 5x\right)=15\sec^4 5x-15\sec^2 5x$. [3]

(ii) Use your answers to part **(i)**, find $\int \sec^4 5x\, dx$. [4]

6 **(i)** Given that $y=\frac{3x}{\sqrt{5-4x}}$, express $\frac{dy}{dx}$ in the form $\frac{ax+b}{\sqrt{(5-4x)^n}}$ where a, b and n are real constants. [4]

(ii) Hence find the equation of the normal to the curve $y=\frac{3x}{\sqrt{5-4x}}$ at the point on the curve where $x=1$. [2]

[Turn over

7 **(i)** On the same diagram, sketch the graphs of $y = 16x^{\frac{5}{3}}$ and $y = \frac{9}{\sqrt[3]{x}}$ for $x > 0$. [3]

(ii) Find the $x-$coordinate of the intersection point of the two graphs. [2]

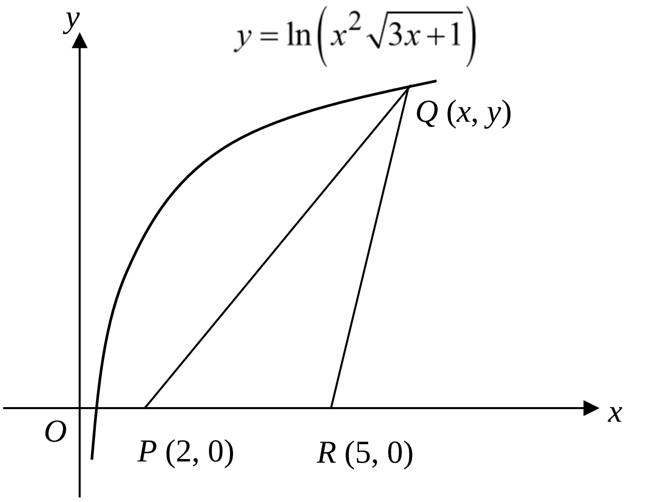
8 Given that $y = \text{cosec}\, x \tan x$,

(i) show that $\frac{dy}{dx} = \sin x \sec^2 x$, and [2]

(ii) determine where y is decreasing for $0 \leq x \leq 2\pi$. [2]

[Turn over

9 The diagram shows the curve $y = \ln\left(x^2\sqrt{3x+1}\right)$ and three points $P(2,\ 0)$, $Q(x,\ y)$ and $R(5,\ 0)$. The point $Q(x,\ y)$ lies on the curve.



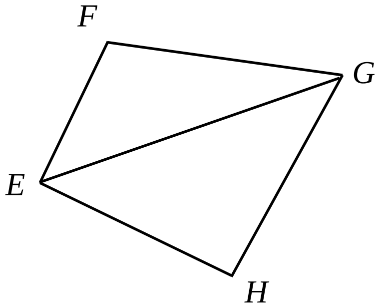
(i) Show that the area, A units^2, of the triangle PQR is given by

$A = 3\ln x + \frac{3}{4}\ln(3x+1)$. [2]

(ii) Given that x is increasing at a rate of 0.2 units/s, find the rate at which the area, A, is changing at the instant when $x = 15$ units. [3]

[Turn over

10



EFGH is a plot of land that comprises two smaller plots, triangle *EFG* and triangle *EGH*. *EF* and *EH* are perpendicular, angle $FEG = \theta$, $EH = 42$ m, $EG = 55$ m and $EF = 48$ m.

(i) Show that the area, A m^2, of *EFGH* can be expressed as $A = 1320\sin\theta + 1155\cos\theta$. [2]

(ii) Express A in the form in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the value of θ if the area is 1231 m^2. [2]

[Turn over

11 **(a)** The variables x and y are related in such a way that when $\frac{x}{y}$ is plotted against $\frac{1}{x}$, a straight line is obtained. The line passes through (2, 9) and (5, 3). Find an expression for y in terms of x. [4]

(b) The table shows experimental values of two variables, *x* and *y*.

x	2	4	6	8
y	8.48	5.99	4.90	4.24

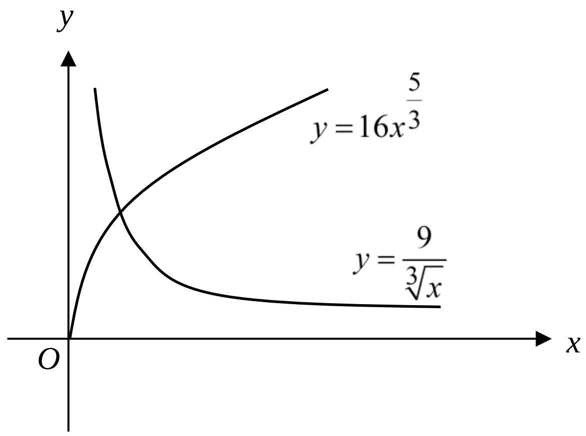
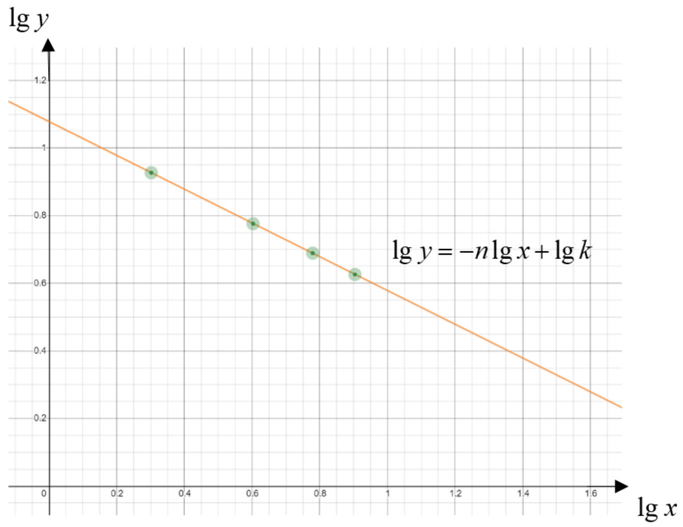
It is known that *x* and *y* are related by the equation $x^n y = k$, where *n* and *k* are constants. Draw a suitable straight line graph to represent the above data and use it to estimate the values of *n* and *k*. [6]

[Turn over

12 Solve the equation $3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)$, giving the principal values of x, in radians. [5]

End of paper

Answer key for AM P1

1	**(i)** $\alpha^2+\beta^2=8$ (ii) $\frac{\alpha^2+\beta^2}{\alpha\beta}=16$ (iii) $x^2+20x+37=0$	**2**	(i)The remaining mass after 30 days is 14.2g. (ii) The number of days required is 28 days. (iii) As $t\to\infty$, $30\,e^{-0.025t}\to\infty$, the value of m approaches 0.
3	**(a)** $a=\frac{1}{4}$ **(b)** $-\frac{3003}{32}$ *or* $-93\frac{27}{32}$	**4**	**(i)** $y=\frac{9}{2}x+15$ **(ii)** $(-4,-3)$ **(iii)** $y=-\frac{2}{9}x+\frac{5}{6}$ (iv) 68 units^2 (v) 76.0° (1 d.p.)
5	**(ii)** $\frac{1}{15}\tan^3 5x+\frac{1}{5}\tan 5x+C_2$		
6	**(i)** $\frac{15-6x}{\sqrt{(5-4x)^3}}$ **(ii)** $9y+x=28$	**7**	**(i)**  **(ii)** $x=\frac{3}{4}$ $(x>0)$
8	**(ii)** For a decreasing function, $\frac{dy}{dx}<0$ For $0\le x\le 2\pi$, $\sec^2 x>0,\ x\ne\frac{\pi}{2},\frac{3\pi}{2}$, $\sin x<0$ for $\pi<x<2\pi$ Hence, y is decreasing for $\pi<x<2\pi,\ x\ne\frac{3\pi}{2}$		
9	**(ii)** The area is increasing at 0.0498 units^2/s.		
10	**(ii)** $A\approx\sqrt{3076425}\sin(\theta+41.2°)$ or $A\approx 165\sqrt{113}\sin(\theta+41.2°)$ **(iii)** $\theta=3.4°$		
12	$\cot x=-\frac{1}{3}$ $\cot x=2$ $\tan x=-3$ or $\tan x=\frac{1}{2}$ $x=-1.25$ $x=0.464$	**11**	**(i)** $y=\frac{x^2}{13x-2}$ (ii)  $k=12.0$ $(11.5-12.6)$ $n=0.500$ $(0.45-0.55)$