Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9758/01

Paper 1

September 2019

3 hours

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Total
Score													
Max Score	**4**	**5**	**6**	**7**	**7**	**7**	**8**	**9**	**10**	**13**	**12**	**12**	**100**

This question paper consists of **26** printed pages and **2** blank pages.

1 A confectionary bakes 20 banana cakes, 50 chocolate cakes and 30 durian cakes every day. The total price of 1 banana cake, 1 chocolate cake and 1 durian cake is \$29.50. On a particular day, at 7 pm, the confectionary has collected \$730 from the sales of the cakes, and there were half the banana cakes, one-tenth of the chocolate cakes and one third of the durian cakes left. In order to sell as many cakes as possible, all cakes were discounted by 40% from their respective selling price from 7 pm onwards. By closing time, all the cakes were sold and the total revenue for the entire day was \$880. Determine the selling price of each type of cake before discount. [4]

2 **(a)** Without using a calculator, solve $\dfrac{30-11x}{x^2-9} \le -2.$ [3]

(b) Solve $(a-3bx^2)e^{ax-bx^3} < 0,$ where a and b are positive constants. [2]

3 The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ with respect to origin O, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel.

(i) Given that B lies on the line segment AC such that $\overrightarrow{BC} = 5\mathbf{b} - \mu\mathbf{a}$, find the value of μ. Hence find $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) The point N is the midpoint of OC. The line segment AN meets OB at point E. Find the position vector of E. [4]

4 The function f is defined as follows:

$$\mathrm{f}(x) = x + \frac{1}{x-a}, \quad a < x \le b$$

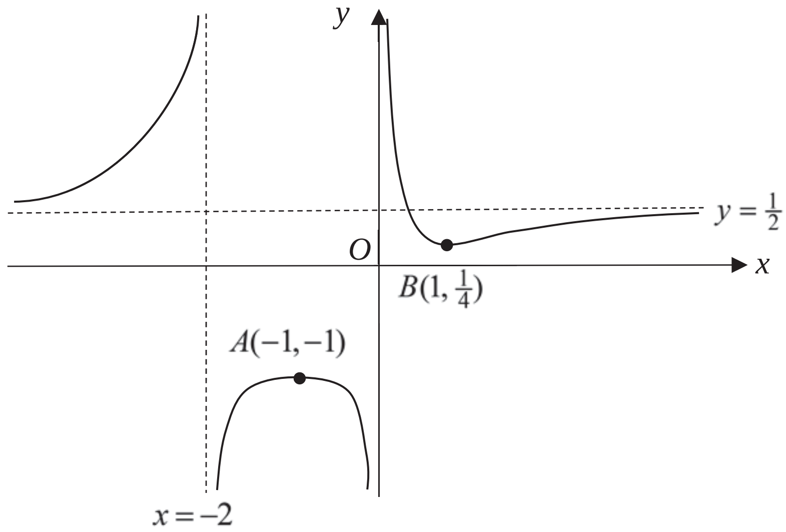
where a is a positive constant.

(i) Given that f^{-1} exist, show that $b \le a+1$. [2]

(ii) Given that $a = 1$ and $b = 2$, find $\mathrm{f}^{-1}(x)$ and the domain of f^{-1}. [5]

5 **(a)** Describe a sequence of two transformations that maps the graph of $y = \ln\left(\dfrac{x^2}{x+1}\right)$ onto the graph of $y = \ln\left(\dfrac{2x+1}{4x^2}\right)$. [2]

(b) The diagram below shows the graph of $y = \mathrm{f}(x)$. It has a maximum point at $A(-1,-1)$ and a minimum point at $B(1, \frac{1}{4})$. The graph has asymptotes $y = \frac{1}{2}$, $x = 0$ and $x = -2$.



Sketch, on separate diagrams, the graphs of

(i) $y = \mathrm{f}(2-x)$, [2]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

stating clearly the equations of any asymptotes, coordinates of any points of intersection with both axes and the points corresponding to *A* and *B*.

6 The sequence of complex numbers $\{w_n\}$ are defined as follows

$$w_n = \frac{\left[1+(n-1)\mathrm{i}\right]\left[1+(n+1)\mathrm{i}\right]}{(1+n\mathrm{i})^2} \text{ for } n \in \mathbb{Z}^+.$$

(i) Show that $\arg(w_n) = p\left[\arg(1+(n-1)\mathrm{i})\right] + q\left[\arg(1+n\mathrm{i})\right] + r\left[\arg(1+(n+1)\mathrm{i}\right]$, where p, q and r are constants to be determined. [1]

Consider a related sequence $\{z_n\}$ where $z_n = w_1 w_2 \dots w_n$, the product of the first n terms of the above sequence.

(ii) Use the method of differences to show that $\arg z_n = -\frac{1}{4}\pi - \arg(1+n\mathrm{i}) + \arg[1+(n+1)\mathrm{i}]$. [4]

(iii) Deduce the limit of $\arg(z_n)$ as $n \to \infty$. Hence write down a linear relationship between $\text{Re}(z_n)$ and $\text{Im}(z_n)$ as $n \to \infty$. [2]

7 A curve C has parametric equations $x = 4\sin 2\theta - 2,\ y = 3 - 4\cos 2\theta$ for $0 \le \theta < \pi$.

(i) Find a cartesian equation of C. Give the geometrical interpretation of C. [3]

(ii) P is a point on C where $\theta = \frac{3}{8}\pi$. The tangent at P meets the y-axis at the point T and the normal at P meets the y-axis at the point N. Find the exact area of triangle NPT. [5]

8 The equations of two planes P_1 and P_2 are $x - 2y + 3z = 4$ and $3x + 2y - z = 4$ respectively.

(i) The planes P_1 and P_2 intersect in a line L. Find a vector equation of L. [2]

The equation of a third plane P_3 is $5x - ky + 6z = 1$, where k is a constant.

(ii) Given that the three planes have no point in common, find the value of k. [2]

Use the value of k found in part **(ii)** for the rest of the question.

(iii) Given Q is a point on L meeting the x-y plane, find the shortest distance from Q to P_3. [3]

(iv) By considering the plane containing Q and parallel to P_3 or otherwise, determine whether the origin O and Q are on the same or opposite side of P_3. [2]

9 It is given that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\mathrm{e}^{-y}-1$ and that $y=0$ when $x=0$.

(i) (a) Show that $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-\left(a+b\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, where a and b are constants to be determined. [2]

(b) Hence, find the first three non-zero terms in the Maclaurin series expansion for y. [2]

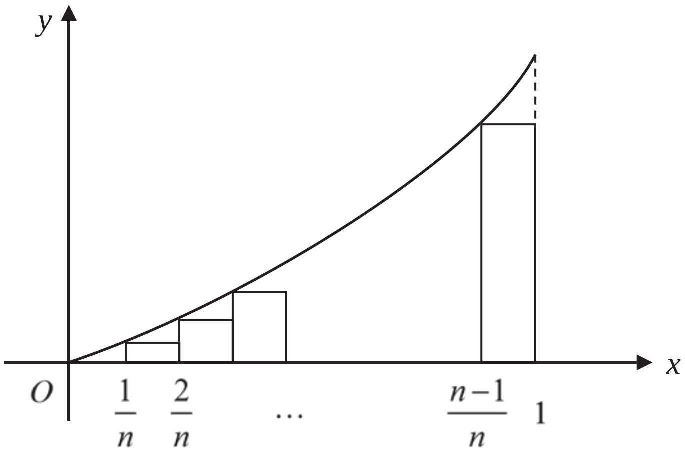
(ii) Find the particular solution of the differential equation, giving your answer in the form $y = \mathrm{f}(x)$. [4]

(iii) Denoting the answer in **(i)(b)** as $\mathrm{g}(x)$, for $x \geq 0$, find the set of values of x for which the value of $\mathrm{g}(x)$ is within ± 0.05 of the value of $\mathrm{f}(x)$. [2]

10 **(a)** Find $\int \frac{x}{\sqrt{2x-1}}\,\mathrm{d}x.$ [3]

(b) Using the substitution $t = \tan x,$ find $\int \frac{1}{4\cos^2 x + 9\sin^2 x}\,\mathrm{d}x.$ [4]

(c)



The diagram above shows part of the graph of $y = x^2 + 3x$, with rectangles approximating the area under the curve from $x = 0$ to $x = 1$. The area under the curve may be approximated by the total area, A, of $(n-1)$ rectangles each of width $\frac{1}{n}$. Given that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$, show that $A = \frac{(n-1)(11n-1)}{6n^2}$.

Explain briefly how the value of $\int_0^1 x^2 + 3x \, dx$ can be deduced from this expression, and hence find this value exactly without integration. [6]

11 The department of statistics of a country has developed two mathematical functions to analyse the foreign worker policy. The first function f models the amount of strain to the country's infrastructure (housing, transportation, utilities and access to medical care etc.) based on the number of foreign workers allowed into the country and is defined as follows:

$$\mathrm{f}(x) = (x-5)^3 + 200, \quad 0 \le x \le 15$$

where x denotes the number of foreign workers, in ten thousands, allowed into the country and $\mathrm{f}(x)$ denotes the amount strain to the country's infrastructure.

The second function g models the happiness index, from 0 (least happy) to 1 (most happy), of the country's local population based on the amount of strain to the country's infrastructure and is defined as follows:

$$\mathrm{g}(w) = \ln\left(\mathrm{e} - \frac{w}{1000}\right), \quad 0 \le w \le 1000(\mathrm{e}-1)$$

where w denotes the amount of strain to the country's infrastructure and $\mathrm{g}(w)$ denotes the happiness index of the country's local population.

(i) The composite function gf models the happiness index based on the number of foreign workers, show that this function exists. [2]

(ii) Find range of values for the happiness index of the country's local population if its government plans to allow 70,000 to 110,000 foreign workers into the country. [3]

(iii) Determine whether the happiness index increases or decreases as x increases. [3]

A third function h models the gross domestic product (GDP) of the country based on the number of foreign workers (in ten thousands), x, allowed into the country and is defined as follows:

$$\mathrm{h}(x) = 400 - (x-10)^2, \quad 0 \leq x \leq 15$$

where $\mathrm{h}(x)$ denotes the GDP in billions of dollars.

(iv) Find the range of values for the GDP if the government plans to have a happiness index from 0.7 to 0.9 in order to secure an electoral win for the coming elections. [4]

12 In a particular chemical reaction, every 2 grams of U and 1 gram of V are combined and converted to form 3 grams of W. Let u, v and w denote the mass (in grams) of U, V and W respectively present at time t (in minutes). According to the law of mass action, the rate of change of w with respect to t is proportional to the product of u and v. Initially, $u = 40$, $v = 50$ and $w = 0$.

(i) Show that $\dfrac{\mathrm{d}w}{\mathrm{d}t} = k(w-60)(w-150)$, where k is a positive constant. [3]

It is observed that when $t = 5,\ w = 10$.

(ii) Find w when $t = 20,$ giving your answer to two decimal places. [7]

(iii) What happens to w for large values of t? [2]