1 Express $\dfrac{12}{x+1}-(7-x)$ as a single simplified fraction. [1]

Without using a calculator, solve $\dfrac{12}{x+1}\leq 7-x$. [3]

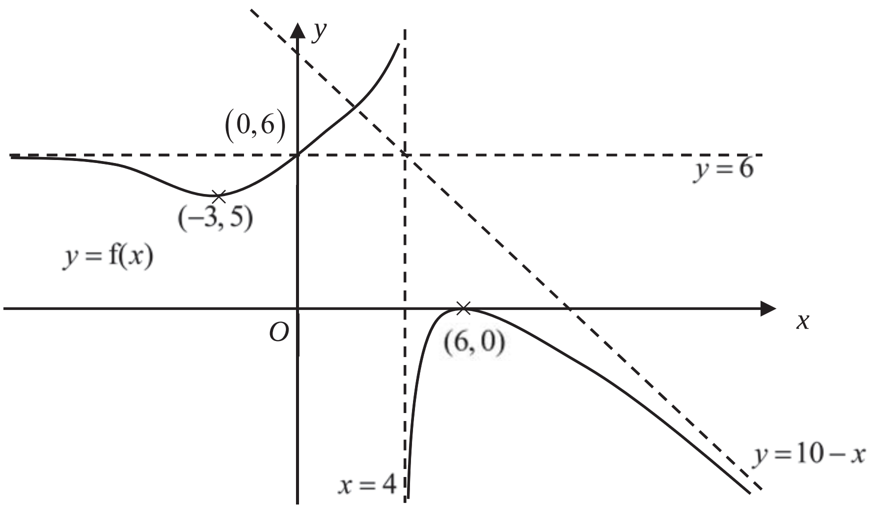
2 **(i)** Find $\dfrac{d}{dx}\tan^{-1}\left(x^2\right)$. [1]

(ii) Hence, or otherwise, evaluate $\int_0^1 x\tan^{-1}\left(x^2\right)dx$ exactly. [3]

3 **(i)** Find $\dfrac{d}{dx}\left(3x^2 2^x\right)$. [2]

(ii) Find the equation of the tangent to the curve $y=3x^2 2^x$ at the point where $x=1$, giving your answer in exact form. [3]

4 The graph for $y=f(x)$ is given below, where $y=10-x$, $y=6$ and $x=4$ are asymptotes. The turning points are $(-3,5)$ and $(6,0)$, and the graph intersects the y-axis at $(0,6)$.



On separate diagrams, sketch the graphs of

(i) $y=f(|x|)$, [3]

(ii) $y=\dfrac{1}{f(x)}$. [3]

5 Referred to the origin O, points P and Q have position vectors $3\mathbf{a}$ and $\mathbf{a}+\mathbf{b}$ respectively. Point M is a point on QP extended such that $PM:QM$ is 2:3.

(i) Find the position vector of point M in terms of **a** and **b**. [2]

(ii) Find $\overrightarrow{PQ}\times\overrightarrow{OM}$ in terms of **a** and **b**. [3]

(iii) State the geometrical meaning of $\dfrac{\left|\overrightarrow{PQ}\times\overrightarrow{OM}\right|}{\left|\overrightarrow{PQ}\right|}$. [1]

6 A curve C has equation $y=\mathrm{f}(x)$, where the function f is defined by

$$\mathrm{f}: x \mapsto \frac{12-3x}{x^2+4x-5}, \quad x\in\mathbb{R}, x\neq -5, x\neq 1.$$

(i) Find algebraically the range of f. [3]

(ii) Sketch C, indicating all essential features. [4]

(iii) Describe a pair of transformations which transforms the graph of C on to the graph of $y=\dfrac{9-x}{x^2-6x}$. [2]

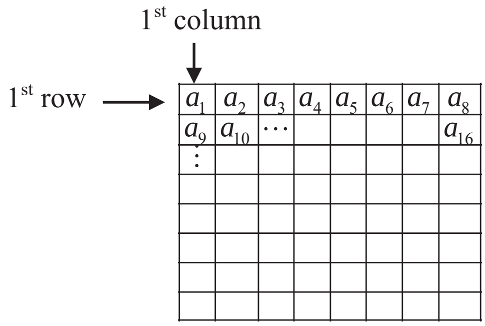
7 Given that $\sin^{-1} y=\ln(1+x)$, where $0<x<1$, show that $(1+x)\dfrac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{1-y^2}$. [2]

(i) By further differentiation, find the Maclaurin expansion of y in ascending powers of x, up to and including the term in x^2. [4]

(ii) Use your expansion from **(i)** and integration to find an approximate expression for $\displaystyle\int \frac{\sin(\ln(1+x))}{x}\,\mathrm{d}x$. Hence find an approximate value for $\displaystyle\int_0^{0.5} \frac{\sin(\ln(1+x))}{x}\,\mathrm{d}x$. [3]

[Turn over

8 **(a)** A sequence of numbers $a_1, a_2, a_3, \ldots, a_{64}$ is such that $a_{n+1} = a_n + d$, where $1 \le n \le 63$ and d is a constant. The 64 numbers fill the 64 squares in the 8×8 grid in such a way that a_1 to a_8 fills the first row of boxes from left to right in that order. Similarly, a_9 to a_{16} fills the second row of boxes from left to right in that order.



Given that the sums of the numbers in the **first row** and in the **third column** are 58 and 376 respectively, find the values of a_1 and d. [4]

(b) A geometric series has first term *a* and common ratio *r*, where *a* and *r* are non-zero. The sum to infinity of the series is 2. The sum of the six terms of this series from the 4th term to the 9th term is $-\dfrac{63}{256}$. Show that $512r^9 - 512r^3 - 63 = 0$.

Find the two possible values of *r*, justifying the choice of your answers. [5]

9 One of the roots of the equation $z^3 - az - 66 = 0$, where *a* is real, is *w*.

(i) Given that $w = b - \sqrt{2}\mathrm{i}$, where *b* is real, find the exact values of *a* and $\dfrac{w}{w^*}$. [6]

(ii) Given instead that $w = r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0,\ -\pi < \theta < -\dfrac{3\pi}{4}$, find $\left|aw^2 + 66w\right|$ and $\arg\left(aw^2 + 66w\right)$ in terms of *r* and θ. [4]

10 The point *M* has position vector relative to the origin *O*, given by $6\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$. The line l_1 has equation $x - 7 = \dfrac{y}{3} = \dfrac{z+2}{-2}$, and the plane π has equation $4x - 2y - z = 30$.

(i) Show that l_1 lies in π. [2]

(ii) Find a cartesian equation of the plane containing l_1 and *M*. [3]

The point *N* is the foot of perpendicular from *M* to l_1. The line l_2 is the line passing through *M* and *N*.

(iii) Find the position vector of *N* and the area of triangle *OMN*. [5]

(iv) Find the acute angle between l_2 and π, giving your answer correct to the nearest $0.1°$. [3]

11 [It is given that the volume of a cylinder with base radius r and height h is $\pi r^2 h$ and the volume of a cone with the same base radius and height is a third of a cylinder.]

A manufacturer makes double-ended coloured pencils that allow users to have two different colours in one pencil. The manufacturer determines that the shape of each coloured pencil is formed by rotating a trapezium $PQRS$ completely about the x-axis, such that it is a solid made up of a cylinder and two cones. The volume, V cm^3, of the coloured pencil should be as large as possible.

It is given that the points P, Q, R and S lie on the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a and b are positive constants. The points R and S are $(-a,0)$ and $(a,0)$ respectively, and the line PQ is parallel to the x-axis.

(i) Verify that $P(a\cos\theta,\ b\sin\theta)$, where $0<\theta<\frac{\pi}{2}$, lies on the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Write down the coordinates of the point Q. [2]

(ii) Show that V can be expressed as $V=k\pi\sin^2\theta(2\cos\theta+1)$, where k is a constant in terms of a and b. [3]

(iii) Given that $\theta=\theta_1$ is the value of θ which gives the maximum value of V, show that θ_1 satisfies the equation $3\cos^2\theta+\cos\theta-1=0$. Hence, find the value of θ_1. [4]

At $\theta=\frac{\pi}{6}$, the manufacturer wants to change one end of the coloured pencil to a rounded-end eraser. The eraser is formed by rotating the arc PS completely about the x-axis.

(iv) Find the volume of the eraser in terms of a and b. [3]

12 A ball-bearing is dropped from a point O and falls vertically through the atmosphere. Its speed at O is zero, and t seconds later, its velocity is v m s^{-1} and its displacement from O is x m. The rate of change of v with respect to t is given by $10-0.001v^2$.

(i) Show that $v=100\left(\frac{e^{\frac{t}{5}}-1}{e^{\frac{t}{5}}+1}\right)$. [4]

(ii) Find the value of v_0, where v_0 is the value approached by v for large values of t. [1]

(iii) By using chain rule, form an equation relating $\frac{dx}{dt}$, $\frac{dv}{dt}$ and $\frac{dv}{dx}$. Given that $v=\frac{dx}{dt}$, form a differential equation relating v and x. Show that

$$v=100\sqrt{1-e^{-\frac{x}{500}}}.$$ [5]

(iv) Find the distance of the ball-bearing from O after 5 seconds, giving your answer correct to 2 decimal places. [3]

[Turn over