2022 H2 Maths Prelim Paper 2 Markers Report

Qn	Solutions	Comments
1(i)	$a \in \mathbb{R}/\{0\}$	Many students were able to quote the condition for an inverse function to exist but failed to realise that when $a=0$, the line becomes a horizontal straight line, which is not one-one.
(ii)	y = fg(x)= 1 – 2g(x) Reflection of the graph of y = g(x) in the x-axis followed by scaling by scale factor 2 parallel to the y-axis followed by translation 1 unit in the positive y-direction. **OR** Translation of the graph of $y=\mathrm{g}(x)$ by $\frac{1}{2}$ unit in the negative y-direction followed by reflection in the x – axis followed by scaling by scale factor 2 parallel to the y-axis. **OR** Translation of the graph of $y=\mathrm{g}(x)$ by $\frac{1}{2}$ unit in the negative y-direction followed by scaling by scale factor 2 parallel to the y-axis followed by reflection in the x – axis.	Better performing students realized that instead of substituting the rule of g(x) into the composite function directly, they could just describe a series of transformation to get from y = g(x) to y = 1 – 2g(x). Students should avoid using words such as ‘flip’ or ‘transform’ to describe any transformations. Instead, to use ‘reflect’, ‘translate’ and ‘scale’. Do also note that scale factor is a positive numerical value and do not contain x in it.
(iii)	$\mathrm{gg}(\boldsymbol{x})=\begin{cases}7-(\boldsymbol{x}^2+3) & \text{for} \quad 0\le \boldsymbol{x}\le 2\\ 7-(7-\boldsymbol{x}) & \text{for} \quad 2<\boldsymbol{x}<5\\ (7-\boldsymbol{x})^2+3 & \text{for} \quad 5\le \boldsymbol{x}\le 7\end{cases}$ $\therefore \mathrm{gg}(\boldsymbol{x})=\begin{cases}4-\boldsymbol{x}^2 & \text{for} \quad 0\le \boldsymbol{x}\le 2\\ \boldsymbol{x} & \text{for} \quad 2<\boldsymbol{x}<5\\ \boldsymbol{x}^2-14\boldsymbol{x}+52 & \text{for} \quad 5\le \boldsymbol{x}\le 7\end{cases}$	This part of the question tests students on the condition for a composite function to exist. For gg to exist, the range of g $\subseteq$ domain of g. For the answer for $5\le \boldsymbol{x}\le 7$, since the range of g is $[0,2]$ (refer to the function g given in the qns whose rule is $2<\boldsymbol{x}\le 7$), the rule $y=7-x$ will be put into ‘x’ of the rule $y=x^2+3$, since its domain is $[0,2]$. For students who put $y=7-x$ into the ‘x’ of $y=7-x$, you may wish to note that range of g = $[0,2]\subsetneq$ domain of g = $(2,7]$.
2(i)	$y=\dfrac{x+a}{b}-\dfrac{a}{x+b}$ Equation of asymptotes are : $y=\dfrac{x+a}{b}$ and $x=-b$	Some students faced difficulty finding the equation of the oblique asymptote. An easier way to see it would be to rewrite the equation of curve as $y=\dfrac{x}{b}+\dfrac{a}{b}-\dfrac{a}{x+b}$,

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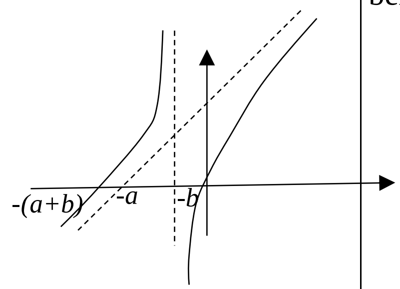
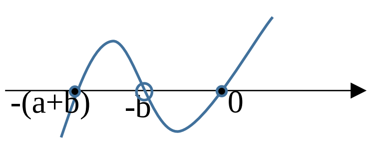
	When $y=0$, $\dfrac{x+a}{b}=\dfrac{a}{x+b}$ $(x+a)(x+b)=ab$ $x^2+x(a+b)=0$ $x(x+(a+b))=0$ $x=0$ or $x=-(a+b)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{b}+\dfrac{a}{(x+b)^2}$ $=\dfrac{(x+b)^2+ab}{b(x+b)^2}>0$ since $a>b>0$, using G.C: 	which resembles the form of $y=mx+c-\dfrac{a}{x+b}$. As the question asks for the equations of asymptotes, just writing $-b$ or $\dfrac{x+a}{b}$ will lead to no mark being awarded.
(i)	**Method 1:** From graph $\therefore x\geq 0$ *or* $-(a+b)\leq x<-b$	Some careless mistakes such as putting b instead of $-b$ or missing out the equal signs at 0 or $-a-b$ were commonly seen. There were also some presentation issues, such as leaving the answer as $0\leq x$. Students may wish to note that a standalone '$,$' is not found in the mathematical notation list provided in the 9758 syllabus. Instead, the use of 'or' is more appropriate for the final answer as either this region $x\geq 0$ or this region $-(a+b)\leq x<-b$ will make the inequality true.
	Method 2: $\dfrac{x+a}{b}\geq\dfrac{a}{x+b}$ $\dfrac{x+a}{b}-\dfrac{a}{x+b}\geq 0$ $\dfrac{x^2+ax+bx+ab-ab}{b(x+b)}\geq 0$ $\dfrac{x^2+ax+bx}{b(x+b)}\geq 0$ But $b>0$, $\therefore \dfrac{x^2+ax+bx}{(x+b)}\geq 0$ $\therefore x(x+a+b)(x+b)\leq 0,\quad x\neq -b$  $\therefore x\geq 0$ *or* $-(a+b)\leq x<-b$	

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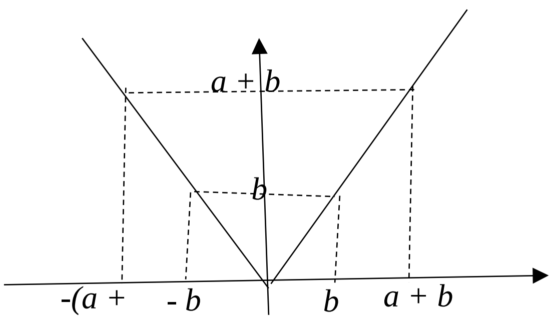
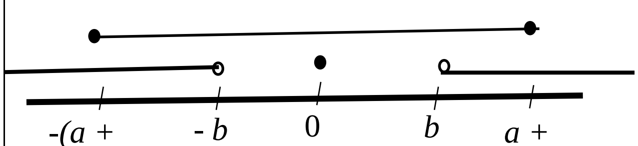
Qn	Solutions	Comments
(ii)	For $\frac{a-\lvert x\rvert}{b} \geq \frac{a}{b-\lvert x\rvert}$ Replace x in (1) with -$\lvert x\rvert$ $\therefore -\lvert x\rvert \geq 0 \;\; or \;\; -(a+b) \leq -\lvert x\rvert < -b$ $\therefore \lvert x\rvert \leq 0$ or $b < \lvert x\rvert \leq (a+b)$	Some common mistakes seen here include: • rejecting $-\lvert x\rvert \geq 0$ immediately, • missing out the equal sign for $a+b$ and $-a-b$ and • poor application of solving inequality with modulus sign. For example, a significant number of students have $-a-b \leq x \leq a+b$ as their answers.
	Method 1:  $\therefore x=0$ or $-(a+b) \leq x < -b$ or $b < x \leq (a+b)$	
	Method 2 $\therefore \lvert x\rvert \leq 0$ or $b < \lvert x\rvert \leq (a+b)$ $\therefore x=0$ or $\lvert x\rvert > b$ and $\lvert x\rvert \leq (a+b)$  $\therefore x=0$ or $-(a+b) \leq x < -b$ or $b < x \leq (a+b)$	
	Method 3 $\frac{a-\lvert x\rvert}{b} \geq \frac{a}{b-\lvert x\rvert} \quad \Rightarrow \quad \frac{a-\lvert x\rvert}{b} - \frac{a}{b-\lvert x\rvert} \geq 0$	

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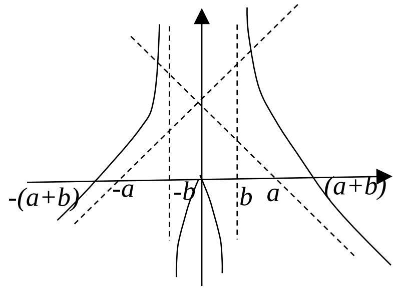
Qn	Solutions	Comments
	Sketch $y=\dfrac{a-\lvert x\rvert}{b}-\dfrac{a}{b-\lvert x\rvert}=\mathrm{f}\left(-\lvert x\rvert\right)$  $\therefore x=0$ or $-(a+b)\le x<-b$ or $b<x\le(a+b)$	
3(i)	$\mathrm{i}z^2-\left(5+\mathrm{i}\right)z+2-6\mathrm{i}=0$ $z=\dfrac{5+\mathrm{i}\pm\sqrt{[-(5+\mathrm{i})]^2-4(\mathrm{i})(2-6\mathrm{i})}}{2\mathrm{i}}$ $=\dfrac{5+\mathrm{i}\pm\sqrt{2\mathrm{i}}}{2\mathrm{i}}$ $=\dfrac{5+\mathrm{i}\pm\left(1+\mathrm{i}\right)}{2\mathrm{i}}$ $=\dfrac{6+2\mathrm{i}}{2\mathrm{i}}$ or $\dfrac{4}{2\mathrm{i}}$ $=1-3\mathrm{i}$ or $-2\mathrm{i}$	There were various methods seen to solve this question, such as completing the square, letting $z=a+b\mathrm{i}$ and proceeding to compare real and imaginary coefficients, etc, but the most efficient way would be to solve it using the quadratic formula and then use the GC to evaluate. For students who convert $a+b\mathrm{i}$ and $a-b\mathrm{i}$ to factors first and compare coefficients with the original question, do note that this method is incorrect as the 2 roots of the quadratic equation do not occur in conjugate pairs, since not all the coefficients are real.
(a) (ii)	$-\mathrm{i}w^2-\left(1-5\mathrm{i}\right)w+2-6\mathrm{i}=0$ Since $w=\mathrm{i}z$, $w=\mathrm{i}\left(1-3\mathrm{i}\right)$ or $\mathrm{i}\left(-2\mathrm{i}\right)$ $=3+\mathrm{i}$ or 2	As this is a hence question, students would have to use the answers from part (i) to make a replacement and then solve for *w*. Solving this part as a fresh new question will warrant zero mark.
(a) (iii)	Since $P(z)$ is a polynomial of degree 4 with real coefficient, hence $1+3\mathrm{i}$ and $2\mathrm{i}$ are also the roots.	The concept tested here is whether students understand that a polynomial with real coefficients would have complex roots in conjugate pairs. Since the 2 roots in part (i) are the roots of $P(z)$, their conjugates would also be roots.

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<table>
<tr><th>Qn</th><th>Solutions</th><th>Comments</th></tr>
<tr><td></td><td>$$\begin{aligned} P(z) &= (z+2\mathrm{i})(z-2\mathrm{i})(z-1-3\mathrm{i})(z-1+3\mathrm{i}) \\ &= (z^2+4)\left((z-1)^2+9\right) \\ &= (z^2+4)(z^2-2z+10) \\ &= z^4-2z^3+14z^2-8z+40 \end{aligned}$$</td><td>Students would also need to be mindful of what they wrote. $P(z)$ is a polynomial in z, not x!</td></tr>
<tr><td>4</td><td>$$\begin{aligned} U_n+U_{n-1} &= \cos(2n+1)\theta+\cos(2(n-1)+1)\theta \\ &= \cos(2n+1)\theta+\cos(2n-1)\theta \\ &= 2\cos\frac{(2n+1)\theta+(2n-1)\theta}{2}\cos\frac{(2n+1)\theta-(2n-1)\theta}{2} \\ &= 2\cos 2n\theta\cos\theta \end{aligned}$$</td><td>Must show the steps on how the MF26 formulas are applied after second line.</td></tr>
<tr><td></td><td>$$\begin{aligned} \sum_{n=1}^{2N}(-1)^{n+1}\cos 2n\theta &= \sum_{n=1}^{2N}(-1)^{n+1}(U_n+U_{n-1}) \\ &= \sum_{n=1}^{2N}(-1)^{n+1}\frac{1}{2\cos\theta}(U_n+U_{n-1}) \\ &= \frac{1}{2\cos\theta}\sum_{n=1}^{2N}(-1)^{n+1}(U_n+U_{n-1}) \\ &= \frac{1}{2\cos\theta}[U_1+U_0 \\ &\qquad -U_2-U_1 \\ &\qquad +U_3+U_2 \\ &\qquad -U_4-U_3 \\ &\qquad \vdots \\ &\qquad +U_{2N-1}+U_{2N-2} \\ &\qquad -U_{2N}-U_{2N-1}] \\ &= \frac{1}{2\cos\theta}[U_0-U_{2N}] \\ &= \frac{1}{2\cos\theta}[\cos\theta-\cos(4N+1)\theta] \\ &= \frac{1}{2}\left(1-\frac{\cos(4N+1)\theta}{\cos\theta}\right) \end{aligned}$$</td><td>Need to re-arrange to get this. Some left out $\frac{1}{2\cos\theta}$.

While doing the Method of Difference, some students mis-read the expression of U_n+U_{n-1}. Instead of starting with U_1+U_0, some started with U_1+U_2 and resulted in cancellation of wrong terms.</td></tr>
</table>

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Qn	Solutions	Comments
	$2n\theta = \frac{n\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$ $\sum_{n=11}^{41}(-1)^{n+1}\cos\frac{n\pi}{3}$ $= \sum_{n=1}^{40}(-1)^{n+1}\cos\frac{n\pi}{3} + (-1)^{41+1}\cos\frac{41\pi}{3} - \sum_{n=1}^{10}(-1)^{n+1}\cos\frac{n\pi}{3}$ $= \frac{1}{2}\left(1 - \frac{\cos(4(20)+1)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right) + \cos\frac{41\pi}{3} - \frac{1}{2}\left(1 - \frac{\cos(4(5)+1)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right)$ $= \frac{1}{2}\left(1 - \frac{\cos(81)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right) + \cos\frac{41\pi}{3} - \frac{1}{2}\left(1 - \frac{\cos(21)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right)$ $= \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$	Most students are able to apply $\sum_{n=11}^{41} = \sum_{n=1}^{41} - \sum_{n=1}^{10}$ Many did not realise that the upper limit in (ii) is 2N, so 41 cannot be = 2N. Hence the need to split to sum from n=1 to n=40 then add 41st term separately. To apply $\sum_{n=1}^{40}$ some didn't realise they 2N=40 to get N = 20 and not n=40, thus $\sum_{n=1}^{40}(-1)^{n+1}\cos\frac{n\pi}{3} = \frac{1}{2}\left(1 - \frac{\cos(4(20)+1)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right)$
5(i)	$x = e^{\cos^{-1}2t}$, $y = (1-4t^2)^{\frac{1}{2}}$ $\frac{dx}{dt} = \frac{-2}{\sqrt{1-4t^2}}e^{\cos^{-1}2t}$, $\frac{dy}{dt} = \frac{1}{2}(-8t)(1-4t^2)^{-\frac{1}{2}} = -4t(1-4t^2)^{-\frac{1}{2}}$ $\frac{dy}{dx} = \frac{-4t(1-4t^2)^{-\frac{1}{2}}}{-2(1-4t^2)^{-\frac{1}{2}}e^{\cos^{-1}2t}}$ $= \frac{2t}{e^{\cos^{-1}2t}}$ As $t = 0, \frac{dy}{dx} = \frac{2t}{e^{\cos^{-1}2t}} = 0$, The tangent to C at $t = 0$ is **parallel to the *x*-axis**.	Some students forgot about the – 2 at the numerator of $\frac{dx}{dt} = \frac{-2}{\sqrt{1-4t^2}}e^{\cos^{-1}2t}$. While applying chain rule, some forgot to write down $e^{\cos^{-1}2t}$ in the $\frac{dx}{dt} = \frac{-2}{\sqrt{1-4t^2}}e^{\cos^{-1}2t}$ Some were able to say that $\frac{dy}{dx} = 0$ but gave wrong conclusion like tangent is a stationary point or // to *y*-axis

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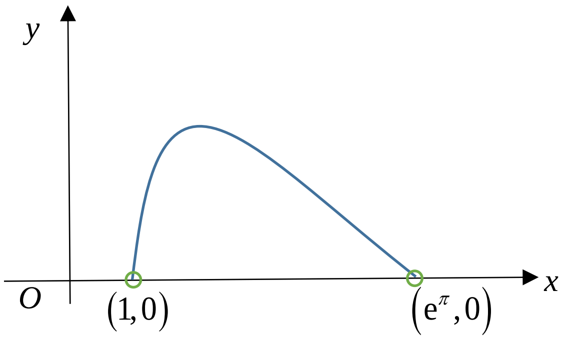
Qn	Solutions	Comments
(ii)		Many students didn't realise that $\frac{-1}{2} < t < \frac{1}{2}$ will result in the open-circles for the end-points on the graph. Only a minority didn't set GC Windows for T-values.
(iii)	Equation of tangent at $P\left(e^{\frac{\pi}{3}}, \frac{\sqrt{3}}{2}\right)$, $t = \frac{1}{4}$, $\frac{dy}{dx} = \frac{1}{2}e^{-\frac{\pi}{3}}$ $y - \frac{\sqrt{3}}{2} = \frac{1}{2}e^{-\frac{\pi}{3}}\left(x - e^{\frac{\pi}{3}}\right)$ $y - \frac{\sqrt{3}}{2} = \frac{1}{2}e^{-\frac{\pi}{3}}\left(x - e^{\frac{\pi}{3}}\right)$ $y = \frac{1}{2}e^{-\frac{\pi}{3}}x + \frac{\sqrt{3}}{2} - \frac{1}{2}$	Some didn't use the coordinate of P to solve for the t-value at that point. Some didn't sub $t = 0.25$ into dy/dx to get the gradient and left the final equation with parameter t.
(iv)	Equation of normal at $Q\left(e^{\cos^{-1}2q}, \left(1-4q^2\right)^{\frac{1}{2}}\right)$, $\frac{dy}{dx} = \frac{2t}{e^{\cos^{-1}2t}} = 0$, $t = q = 0$, Sub $t = q = 0$ into $x = e^{\cos^{-1}2(0)} = e^{\frac{\pi}{2}}$ so $Q\left(e^{\frac{\pi}{2}}, 1\right)$ $\therefore x = e^{\frac{\pi}{2}}$ (Note: the normal is // y-axis)	Some didn't realise that for the normal to be // to y-axis, gradient of the tangent = 0 $\Rightarrow \frac{dy}{dx} = \frac{2t}{e^{\cos^{-1}2t}} = 0 \quad \Rightarrow t = 0$ Some didn't realise that the normal to be // to y-axis has the $x = h$ type of equation

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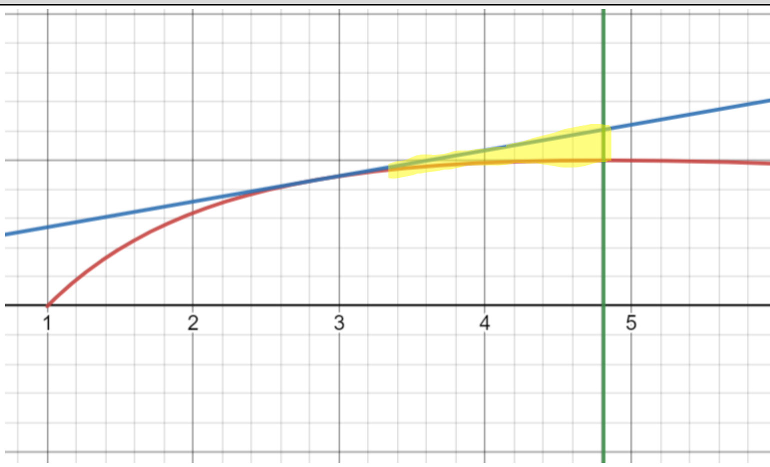
Qn	**Solutions**	**Comments**
(v)	 Required area $=\int_{e^{\frac{\pi}{3}}}^{e^{\frac{\pi}{2}}}\left(\frac{1}{2}e^{-\frac{\pi}{3}}x+\frac{\sqrt{3}}{2}-\frac{1}{2}\right)dx-\int_{e^{\frac{\pi}{3}}}^{e^{\frac{\pi}{2}}}(y)dx$ $=\int_{e^{\frac{\pi}{3}}}^{e^{\frac{\pi}{2}}}\left(\frac{1}{2}e^{-\frac{\pi}{3}}x+\frac{\sqrt{3}}{2}-\frac{1}{2}\right)dx-\int_{\frac{1}{4}}^{0}\left(1-4t^2\right)^{\frac{1}{2}}\frac{-2}{\sqrt{1-4t^2}}e^{\cos^{-1}2t}dt$ $=\int_{e^{\frac{\pi}{3}}}^{e^{\frac{\pi}{2}}}\left(\frac{1}{2}e^{-\frac{\pi}{3}}x+\frac{\sqrt{3}}{2}-\frac{1}{2}\right)dx-\int_{\frac{1}{4}}^{0}-2e^{\cos^{-1}2t}dt$ $=2.03543-1.88372$ $=0.15171$ $=0.152$	Some were unable to tell where the required area is, therefore using the wrong equation for the upper curve. For the parametric part, some students forgot about the $\frac{dx}{dt}$ part of the integration in $\int_{\frac{1}{4}}^{0}\left(1-4t^2\right)^{\frac{1}{2}}\frac{-2}{\sqrt{1-4t^2}}e^{\cos^{-1}2t}dt$.
6 (i)	**Case 1:** All three magnets are different colour. ${}^4C_3=4$ ways ${}^4C_3$: Out of 4 colours, select one 3 colors. **Case 2:** Two identical and one different colour. ${}^4C_2\times{}^2C_1=12$ ways ${}^4C_2$: Out of 4 colours choose 2 colours. ${}^2C_1$: Out of the 2 chosen colours, select one colour to be the repeated colour. Alternatively, ${}^4C_1\times{}^3C_1=12$ ways ${}^4C_1$: Out of 4 colours choose 1 colour to be the repeated colour.	Many students are able to identify the 3 categories of cases. However quite a number of students have difficulty finding the correct number of ways for each case. The most common mistake is not realising that the magnets of the same colour are identical and hence not distinct. For example to find the number of ways to choose 3 magnets of different colour, common mistakes includes writing ${}^{14}C_3$, ${}^5C_1\times{}^4C_1\times{}^3C_1$, etc.

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Qn	Solutions	Comments
	${}^3C_1$: Out of the 3 remaining unchosen colours, select one color to be the magnet with a different colour. **Case 3:** All three magnets identical (same colour). All red or all blue or all orange. (3 ways) Total number of ways $= 4+12+3 = 19$ ways	
6(ii)	Method 1: $\text{P}(\text{1 Red, 2 Blue, 1 Orange}) = \left(\frac{5}{14}\right)\left(\frac{4}{13}\right)\left(\frac{3}{12}\right)\left(\frac{3}{11}\right)\left(\frac{4!}{2!}\right) = \frac{2160}{24024} = \frac{90}{1001}$ $\text{P}(\text{2 Red, 1 Blue, 1 Green}) = \left(\frac{5}{14}\right)\left(\frac{4}{13}\right)\left(\frac{4}{12}\right)\left(\frac{2}{11}\right)\left(\frac{4!}{2!}\right) = \frac{1920}{24024} = \frac{80}{1001}$ $\text{P}(\text{1 Red, 2 Orange, 1 Green}) = \left(\frac{5}{14}\right)\left(\frac{3}{13}\right)\left(\frac{2}{12}\right)\left(\frac{2}{11}\right)\left(\frac{4!}{2!}\right) = \frac{720}{24024} = \frac{30}{1001}$ $\text{P}(\text{3 Orange, 1 Blue}) = \left(\frac{3}{14}\right)\left(\frac{2}{13}\right)\left(\frac{1}{12}\right)\left(\frac{4}{11}\right)\left(\frac{4!}{3!}\right) = \frac{96}{24024} = \frac{4}{1001}$ $\text{P}(\text{\$7 worth of magnets}) = \frac{90+80+30+4}{1001} = \frac{204}{1001}$ Method 2: $\text{P}(\text{1 Red, 2 Blue, 1 Orange}) = \frac{{}^5C_1 \times {}^4C_2 \times {}^3C_1}{{}^{14}C_4} = \frac{90}{1001}$ $\text{P}(\text{2 Red, 1 Blue, 1 Green}) = \frac{{}^5C_2 \times {}^4C_1 \times {}^2C_1}{{}^{14}C_4} = \frac{80}{1001}$ $\text{P}(\text{1 Red, 2 Orange, 1 Green}) = \frac{{}^5C_1 \times {}^3C_2 \times {}^2C_1}{{}^{14}C_4} = \frac{30}{1001}$ $\text{P}(\text{3 Orange, 1 Blue}) = \frac{{}^3C_3 \times {}^4C_1}{{}^{14}C_4} = \frac{4}{1001}$ $\text{P}(\text{\$7 worth of magnets}) = \frac{90+80+30+4}{1001} = \frac{204}{1001}$	Most students are able to identify at least 3 out of the 4 cases for $7 worth of magnets. For method 1, the most common mistake is missing out on either $\left(\frac{4!}{2!}\right)$ or $\left(\frac{4!}{3!}\right)$. Some students also did not realise that the magnets were chosen <u>without</u> replacement. Method 2 is not a commonly used method among students. Some students who used method 2 have problems finding the correct numerator for each case. For example for the case of 1 Red, 2 Blue and 1 Orange, some made the mistake of writing ${}^5C_1 \times \left({}^4C_1 \times {}^3C_1\right) \times {}^3C_1$.

Qn	Solutions	Comments
6 (iii)	No. of ways $=(10-1)!\ \times 2!\ \times {}^{10}C_3 \times 3!$ $=522547200$ $(10-1)!$ refers to arranging 5 red, 4 blue and one group of 2 adjacent green in a circle. $2!$ refers to permutating among the 2 adjacent green magnets. ${}^{10}C_3$ refers to choosing 3 slots among the 10 slots for the 3 orange magnets. 3! Refers to permutating among the 3 orange magnets. <u>Complement method (not recommended for this question):</u> Total no. of arrangements such that 2 green magnets are adjacent $=(13-1)!\ \times 2! = 958003200$ No. of arrangements such that 2 green magnets are adjacent and only 2 orange magnets are together $=(10-1)!\ \times 2! \times \left[\left({}^{10}C_2 \times 2!\right)\times\left({}^{3}C_2 \times 2!\right)\right]$ $=391910400$ $(10-1)!\ \times 2!$ refers to arranging 1 group of 2 green magnets, 4 red and 5 blue magnets in a circle. 2! refers to the group of 2 adjacent orange magnets swopping positions. $\left({}^{10}C_2 \times 2!\right)$ refers to choosing 3 slots among the 10 slots for the one group of 2 adjacent orange magnets and another separate orange magnet. 2! refers to the group of 2 adjacent orange magnets and the separate orange magnet swopping positions. $\left({}^{3}C_2 \times 2!\right)$ refers to choosing 2 orange magnets from the 3 orange magnets to be together. 2! refers to the group of 2 adjacent orange magnets swopping positions. No. of arrangements such that 2 green magnets are adjacent and 3 orange magnets are together $=(11-1)!\ \times 2! \times 3!$ $=43545600$ Required no. of ways $=958003200-391910400-43545600$ $=522547200$	This question is manageable for many students. Common mistakes includes missing out on either 2! or 3!. Another common mistake is students writing $(9-1)!\ \times 2!\ \times {}^{9}C_4 \times 4!$ because they first arrange the 5 red and 4 blue marbles in a circle and then choose 4 slots to insert the 3 orange magnets and 1 group of 2 green magnets. However, this method is incorrect as it is missing out on cases whereby the green magnets could be adjacent to the orange magnets. Some students tried the complement method but with only a few being successful.

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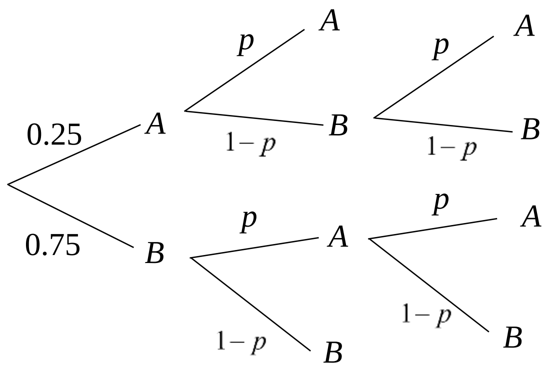
Qn	Solutions	Comments
7 (i)	Let A be the event where Archer wins a game. Let B be the event where Betty wins a game. 	Note that the competition ends once any player wins two consecutive games.
7 (ii)	$\mathrm{P}(\text{Archer wins the competition})$ $= 0.25p + 0.25(1-p)p + 0.75p^2$ $= 0.25p + 0.25p - 0.25p^2 + 0.75p^2$ $= 0.5p + 0.5p^2$ $= 0.5p(1+p)$	There are only 3 possible outcomes for Archer to win the competition, i.e. WW, WLW, LWW (W: Win L:Lose)
7 (iii)	$\mathrm{P}(\text{Betty won 2nd game} \mid \text{Betty won the competition})$ $= \dfrac{\mathrm{P}(\text{Betty lost 1st and won 2nd \& 3rd game}) + \mathrm{P}(\text{Betty won 1st \& 2nd game})}{\mathrm{P}(\text{Betty won the competition})}$ $= \dfrac{(0.25)(1-p)(1-p) + (0.75)(1-p)}{1 - \mathrm{P}(\text{Archer won the competition})}$ $= \dfrac{(0.25)(1-p)(1-p) + (0.75)(1-p)}{1 - 0.5p(p+1)}$ Since $p = 0.5$, $\mathrm{P}(\text{Betty won 2nd game} \mid \text{Betty won the competition})$ $= \dfrac{(0.25)(1-0.5)(1-0.5) + (0.75)(1-0.5)}{1 - (0.25)(0.5+1)}$ $= \dfrac{(0.25)(0.5)(0.5) + (0.75)(0.5)}{1 - 0.375}$ $= 0.7$	"given that" suggests this is a conditional probability question. Note that the event described in the denominator is the complement of the event in **(ii)**. Hence, the calculation is straightforward.

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Qn	Solutions	Comments
	<u>Alternative method to find P(Betty won the competition)</u> $\mathrm{P(WW)+P(WLW)+P(LWW)}$ $=0.75(1-p)+0.75p(1-p)+0.25(1-p)^2$ $=0.75(0.5)+0.75(0.5)(0.5)+0.25(0.5)^2$ $=0.625$	
7 (iv)	$\begin{array}{\|c\|c\|c\|c\|} \hline w & 0 & 1 & 2 \\ \hline \mathrm{P}(W=w) & \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)=\frac{3}{8} & \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4} & \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{3}{8} \\ \hline \end{array}$	Common mistakes seen were omitting the case where $w=0$ or including the case where $w=3$, which is clearly a misinterpretation of the question.
7 (v)	$\mathrm{E}(W)=(0)\left(\frac{3}{8}\right)+(1)\left(\frac{1}{4}\right)+(2)\left(\frac{3}{8}\right)=1$ $\mathrm{E}(W^2)=(0)^2\left(\frac{3}{8}\right)+(1)^2\left(\frac{1}{4}\right)+(2)^2\left(\frac{3}{8}\right)=\frac{7}{4}=1.75$ $\mathrm{Var}(W)=\mathrm{E}(W^2)-\left[\mathrm{E}(W)\right]^2=1.75-1^2=0.75$	Working must be shown when finding $\mathrm{E}(W^2)$. Writing $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$ are not appropriate here. Students should write the notations according to context.
Q8 (i)	*a* represents for every additional month that a 1-kilogram Cheddar cheese is aged, the price increases by \$*a*.	Students should quote \$*a* instead of *a* units and mention that it is a <u>fixed increase</u> in the price for each additional month. Phrasings like “rate of change…with respect to…”, “rate of increase”, ‘change in price” etc. are vague and should be avoided.
(ii)	(A) 0.9085 (B) 0.9751	Question clearly stated the accuracy required for the answers (i.e. 4 d.p). Giving your answers to 3 s.f. are not acceptable.

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Qn	Solutions	Comments
(iii)	(B) is a better model since the <u>r value is closer to 1</u> as compared to model (A). Thus the data points in model (B) are more clustered along a straight line. Equation of a suitable straight line: $\ln P = 2.5641 + 0.15806\, m$ $\ln P = 2.56 + 0.158\, m$ (3 s.f) [Final answer]	It is not sufficient to claim that r value of (B) is greater than that of (A). The relativity of these values to 1 is more critical. It is not convenient to justify (B) is a better model by describing the behaviour of P as m increases as the scatter diagrams are not readily available. Coefficients of regression line should be given to 3 s.f. for final answer unless otherwise specified.
(iv)	When $m = 14$, $\ln P = 4.776945$ $P = \$\ 118.74$ (to the nearest cent) $m = 14$ is within the sample data range of m ($2.2 \leq m \leq 17.2$), where the linear relationship still holds, thus the use of this model to predict the value of P is appropriate and reliable.	Use accuracy of 5 s.f. for the coefficients of regression line when performing estimations. It is a serious misconception to say that the estimate of P is reliable because the estimate is in the data range of P. In addition, claiming that "the estimate is reliable because it is in the data range" is ambiguous (Is it referring to m or P?)
(v)	$\ln 1000P = 2.56 + 0.158\, m$ (3 s.f)	Note that the notation P, is **unchanged** in the question except that its unit is now given as dollars per gram. To convert to dollars per kilogram, simply multiply P by 1000.

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Q9 (i)	$T \sim \mathrm{N}(144, 25)$ Since $\mathrm{P}(T > 60k) = 0.5$, $60k$ is the mean of the distribution, 144 min. $\therefore k = \dfrac{144}{60} = 2.4$	
(ii)	$T \sim \mathrm{N}(144, 25)$, $X \sim \mathrm{N}(236, 81)$ $0.9X - T \sim \mathrm{N}\left(0.9(236) - (144),\ 0.9^2(81) + (25)\right)$ $0.9X - T \sim \mathrm{N}(68.4,\ 90.61)$ $\mathrm{P}(0.9X - T \le 60) = 0.18877 = 0.189$ (to 3 s.f)	**Note:** ➢ use $0.9X$ or another variable to denote X after 7pm. ➢ $0.9X \sim \mathrm{N}\left(212.4, 0.9^2(81)\right)$ ➢ Since train is faster, it takes a shorter time, hence $\lvert 0.9X - T \rvert$ is NOT necessary. ➢ Qn requires $0.9X - T \le 60$, **not** $T - 0.9X \le 60$, **not** $0.9X - 0.9T \le 60$, **not** $0.9X - T \le 1$. ➢ "$\le$" is required, not "$<$". ➢ $\mathrm{Var}(0.9X - T)$ $= \mathrm{Var}(0.9X) + \mathrm{Var}(T)$ (Plus! Not minus)
(iii)	Required conditional probability $= \dfrac{\mathrm{P}(\text{he takes train and took more than 4hrs, i.e } T > 2.4\text{hrs})}{\mathrm{P}(\text{he takes either train or express bus and took more than 4 hrs})}$ $= \dfrac{0.7\,\mathrm{P}\left(T > (4-1.6)\times 60\right)}{0.7\,\mathrm{P}\left(T > (4-1.6)\times 60\right) + 0.3\,\mathrm{P}(X > 4\times 60)}$ $= \dfrac{0.7(0.5)}{0.7(0.5) + 0.3\,\mathrm{P}(X > 240)}$ (from (i)) $= \dfrac{0.7(0.5)}{0.7(0.5) + 0.3(0.32836)}$ $= 0.780365 = 0.780$ (to 3 s.f)	This question is a conditional probability since it is **given** that the time taken is more than 4 hours. In G.C, when normalcdf gives 0.49999, the probability is actually 0.5 because the x value is the mean of the distribution. Eg. $\mathrm{P}(T > 144) = 0.5$ since 144 is the mean. G.C could not give exact value due to rounding off error. 0.7 or 0.3 was commonly missing.
(iv)	Expected cost = 0.7 (54) + 0.3 (24) = \$45	Note that (iii) and (iv) are not linked.

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10 (i)	To test $H_0 : \mu = 535$ Against $H_1 : \mu < 535$ where μ represents the population mean daily profit of the nasi lemak stall holder.	Note that x, $\bar{x}$, μ_O or μ_1 are **not** acceptable.
(ii)	It is not necessary to assume that the daily profits are normally distributed for the test to be valid since $n = 45$ is large, Central Limit Theorem applies. [Note: By Central Limit Theorem, **sample means** of the daily profits of sample size 45 will follow a normal distribution approximately.]	Sample size in this case is 45, **not** the number of days in past years. Central Limit Theorem applies, $\bar{X}$ follows a normal distribution approximately, not X.
(iii)	Since H_0 is rejected at 5% level of significance, $z = \dfrac{\bar{x} - 535}{\sqrt{2591/45}} < -1.645$ $\bar{x} < -1.645\left(\sqrt{2591/45}\right) + 535$ $\{ \bar{x} : \bar{x} < 522.52\}$ $\therefore \{ \bar{x} : 0 < \bar{x} < 523\}$ (to 3 s.f)	**Different types of "Variances" (In order of importance)** 1) Population variance : σ^2 ➢ Most useful but usually not available. ➢ $\sigma^2 = 2591$ 2) Unbiased estimate of population variance : s^2 ➢ Quite useful. ➢ Good "substitute" when population variance is not available. ➢ Formulae found in MF26. ➢ $s^2 = \dfrac{n}{n-1} \times$ sample variance 3) Sample variance: No symbol ➢ Most useless. ➢ Cannot be used directly. ➢ Always convert to s^2.
(iv)	Unbiased estimate for population variance is $s^2 = \dfrac{45}{45-1} \times$ sample variance $= \dfrac{45}{45-1} \times 2008 = 2053.6364$ Under H_0, $\bar{X} \sim N\left(535, \dfrac{2053.6364}{45}\right)$ approx. by Central Limit Theorem since $n = 45$ is large Value of test statistic, $z = \dfrac{520 - 535}{\sqrt{2053.6364/45}} = -2.22$ p - value = 0.013195 < 0.05 $\therefore$ Reject H_0. There is sufficient evidence at the 5% level of significance to conclude that the population mean daily profit is less than \$535.	

(v)	If H_0 is not rejected, $p-\text{value} > \frac{\alpha}{100}$ $0.013195 > \frac{\alpha}{100}$ $\alpha < 1.3195$ The largest level of significance is 1.31%. (to 2 d.p) (1.32% is also acceptable.)	Level of significance refers to α %, not $\frac{\alpha}{100}$.
(vi)	To test $H_0 : \mu = 535$ Against $H_1 : \mu \neq 535$ at 8% level of significance Under H_0, $\bar{X} \sim \text{N}\left(535, \frac{2591}{n}\right)$ approx. by Central Limit Theorem since n is large. Value of test statistic, $z = \frac{526-535}{\sqrt{2591/n}}$ In order not to reject H_0, $-1.750686 < \frac{526-535}{\sqrt{2591/n}} < 1.750686$ $-9.901465 < \sqrt{n} < 9.901465$ $0 < n < 98.039016$ Greatest value of n is 98. [Note: In inequalities, before squaring both sides, it is necessary to check whether the inequality "make sense". For example, if $\frac{-9}{\sqrt{2591/n}} > 1.750686$, this doesn't make sense for any real values of n since a negative value (LHS) cannot be larger than a positive value (RHS). It is INCORRECT to square both sides to obtain n. Likewise, if $\frac{-9}{\sqrt{2591/n}} < 1.750686$, this inequality is true for any real values of n, do not square both sides to obtain a range of values of n blindly.]	Note that this is a 2-tail test where $H_1 : \mu \neq 535$ at 8% level of significance. Some common incorrect critical values: 1) 1.4050716, due to 1 tail test. 2) 0.1004337, due to invNorm center 0.08. InvNorm center is referring to the area in the middle. In this case, it should be InvNorm center 0.92 instead. Alternatively, invNorm left 0.04 or invNorm right 0.04. **DO NOT WRITE:** $\frac{526-535}{\sqrt{2591/n}} \sim N(0,1)$ The correct way is $\frac{\bar{X}-535}{\sqrt{2591/n}} \sim N(0,1)$ Errors were commonly seen when solving the inequalities algebraically.

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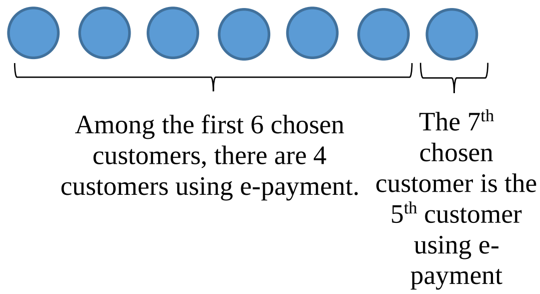
11 (a) (i)	 Method 1: Let W be the random variable the number of customers out of first 6 chosen customers who uses e-payment at a hawker stall. $W \sim \mathrm{B}(6, 0.25)$ $\text{Required Probability} = \mathrm{P}(W = 4) \times 0.25$ $= 0.00824$ (3.s.f) Method 2: $\text{Required Probability} = (0.25)^4 (0.75)^2 \left(\frac{6!}{4!2!}\right) \times 0.25$ $= 0.00824$ (3.s.f)	There are many successful attempts for this question for those who uses method 1. For method 2, the most common mistake is the missing term $\left(\frac{6!}{4!2!}\right)$. $\left(\frac{6!}{4!2!}\right)$ is necessary to account for the various permutations of the first 6 customers whereby 4 of them uses e-payments and 2 of them do not.
(a) (ii)	Let C be the random variable the number of customers out of 40 customers who uses e-payment at a hawker stall. $C \sim \mathrm{B}(40, 0.25)$ $\mathrm{P}(C \le 10) = 0.583904$ Let A be the random variable the number of customers out of 40 customers who uses e-payment at a hawker stall. $A \sim \mathrm{B}(30, 0.583904)$ $\mathrm{P}(A \ge 15) = 1 - \mathrm{P}(A \le 14)$ $= 1 - 0.1322406$ $= 0.867759$ $= 0.868$ (3s.f)	This part is successfully attempted by most students. It is heartening to observe that many student defined the random variables and distributions used. Some students misinterpreted or misread the question and ended up writing things such as $\mathrm{P}(C < 10)$, $\mathrm{P}(C \ge 10)$, $\mathrm{P}(A > 15)$ $\mathrm{P}(A \le 15)$, etc Note that $\mathrm{P}(A \ge 15) \ne 1 - \mathrm{P}(A \le 15)$ $\mathrm{P}(A \ge 15) \ne 1 - \mathrm{P}(A < 14)$

(b) (i)	A random sample in this context means that each hawker has an equal chance of being selected for the sample and the selection of one hawker is independent of any other hawkers.	This part is not well explained by many students. Note that each hawker has an equal chance of being selected for the sample. Quite a number of students thought that only hawkers who uses the Foodgowhere has an equal chance of being selected for the sample. Another common mistake is writing that the probability of a hawker being selected for the sample is independent of other hawkers. Some students did not answer the question but instead wrote about the conditions necessary for a binomial distributions to be applicable.
(b) (ii)	$X \sim \mathrm{B}(n, p)$ Since $\mathrm{E}(X) = 3.96$, $\therefore np = 3.96$ --------------(1) Since $\mathrm{P}(X \le 1) = 0.05303$, $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.05303$ ${}^{n}C_{0}\, p^{0} (1-p)^{n} + {}^{n}C_{1}\, p^{1} (1-p)^{n-1} = 0.05303$ $(1-p)^{n} + np(1-p)^{n-1} = 0.05303$ --------------(2) Sub. (1) into (2): $(1-p)^{3.96/p} + (3.96)(1-p)^{3.96/p - 1} = 0.05303$ Using GC, $p = 0.3600375 = 0.360$ (3s.f) $\therefore n = \dfrac{3.96}{0.3600375} = 11$ (nearest integer)	This part is well attempted by most students. Most students are able to write out equation (1). When writing out equation (2), some left out $\mathrm{P}(X = 0)$ for the condition $\mathrm{P}(X \le 1) = 0.05303$. Some students did not know how to solve the 2 equations using GC. Note n is an integer and hence answer for n should not be in 3s.f.

(b) (iii)	$X \sim \mathrm{B}(15, p)$ Since mode $= 5 \Rightarrow$ $\mathrm{P}(X=4) < \mathrm{P}(X=5)$ and $\mathrm{P}(X=5) > \mathrm{P}(X=6)$ Considering $\mathrm{P}(X=4) < \mathrm{P}(X=5)$, ${}^{15}C_4 p^4 (1-p)^{11} < {}^{15}C_5 p^5 (1-p)^{10}$ $(1365)\left(p^4\right)(1-p)^{11} < (3003)\left(p^5\right)(1-p)^{10}$ $\left(p^4\right)(1-p)^{10}\left[1365(1-p)-3003p\right] < 0$ Since $(1-p) > 0$ and $p > 0$, $1365(1-p)-3003p < 0$ $1-p-2.2p < 0$ $p > \frac{5}{16}$ Considering $\mathrm{P}(X=5) > \mathrm{P}(X=6)$, ${}^{15}C_5 p^5 (1-p)^{10} > {}^{15}C_6 p^6 (1-p)^9$ $(3003)\left(p^5\right)(1-p)^{10} > (5005)\left(p^6\right)(1-p)^9$ $\left(p^5\right)(1-p)^9\left[3003(1-p)-5005p\right] > 0$ Since $(1-p) > 0$ and $p > 0$, $3003(1-p)-5005p > 0$ $1-p-\frac{5}{3}p > 0$ $p < \frac{3}{8}$ Combining both results, $\therefore \frac{5}{16} < p < \frac{3}{8}$	Most students are aware that the modal value is 5 and many were able to state the 2 conditions needed for a modal value of 5. Some students only stated one condition while some did not know that we need to combine the results from the 2 conditions. Some attempted to use the GC to solve but ended up only stating one specific value of p whereby the modal value is 5.