ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME

TUTORIAL/ FORM CLASS

INDEX NUMBER

MATHEMATICS 9758/01

Paper 1 **29 August 2019**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of __ printed pages.

Anglo-Chinese Junior College **[Turn Over**

1 The points $A(2,-3)$ and $B(-3,1)$ are on a curve with equation $y = \mathrm{f}(x)$. The corresponding points on the curve $y = \mathrm{f}\big(a(x-b)\big)$ are $A'(7,-3)$ and $B'(-1,1)$. Find the values of a and b. [3]

2 Use differentiation to find the area of the largest rectangle with sides parallel to the coordinate axes, lying above the x-axis and below the curve with equation $y = 44 + 4x - x^2$. [5]

3 Solve the equation $\left|\dfrac{x^2+3x}{x-1}\right| = 2x+3$ exactly. [4]

Hence, by sketching appropriate graphs, solve the inequality $\left|\dfrac{x^2+3x}{x-1}\right| < 2x+3$ exactly. [2]

4 A kite 50 m above ground is being blown away from the person holding its string in a direction parallel to the ground at a rate 5 m per second. Assuming that the string is taut, at the instant when the length of the string already let out is 100 m, find, leaving your answers in exact form,

(i) the rate of change of the angle between the string and the ground, [3]

(ii) the rate at which the string of the kite should be let out, [4]

5 Given that $y = \tan\left(1-\mathrm{e}^{3x}\right)$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = k\mathrm{e}^{3x}\left(1+y^2\right)$, where k is a constant to be determined. By further differentiation of this result, or otherwise, find the first three non-zero terms in the Maclaurin series for $\tan\left(1-\mathrm{e}^{3x}\right)$. [5]

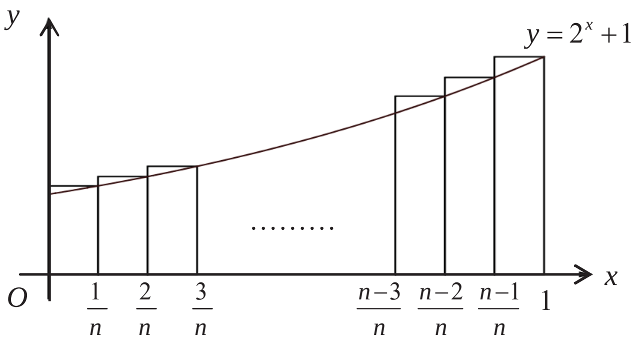
The first two terms in the Maclaurin series for $\tan\left(1-\mathrm{e}^{3x}\right)$ are equal to the first two non-zero terms in the series expansion of $\dfrac{x}{a+bx}$. Find the constants a and b. [3]

6 The diagram below shows the graph of $y = 2^x + 1$ for $0 \le x \le 1$. Rectangles, each of width $\frac{1}{n}$, are also drawn on the graph as shown.

Show that the total area of all n rectangles, S_n, is given by

$$S_n = \frac{2^{\frac{1}{n}}}{n(2^{\frac{1}{n}} - 1)} + 1 .$$ [3]

Find the exact value of $\lim_{n \to \infty} S_n$. [2]



7 **(a)** Find $\int \sin px \cos qx \, dx$ where p and q are positive integers such that $p \ne q$. [2]

(b) Show that $\int x \sin nx \, dx = -\frac{x \cos nx}{n} + \frac{\sin nx}{n^2} + c$ where n is a positive integer and c is an arbitrary constant. [1]

Hence find

(i) $\int_0^{\pi} x \sin nx \, dx$, giving your answers in the form $\frac{k\pi}{n}$ where the possible values of k are to be determined, [2]

(ii) $\int_0^{\frac{\pi}{2}} |x \sin 3x| \, dx$ in terms of π. [3]

8 **Do not use a calculator in answering this question.**

(a) The complex numbers z and w satisfy the following equations

$$w - 2z = 9,$$
$$3w - wz^* = 17 - 30\text{i}.$$

Find w and z in the form $a + b\text{i}$, where a and b are real and $\text{Re}(z) < 0$. [4]

(b) **(i)** Given that $-\text{i}$ is a root of the equation

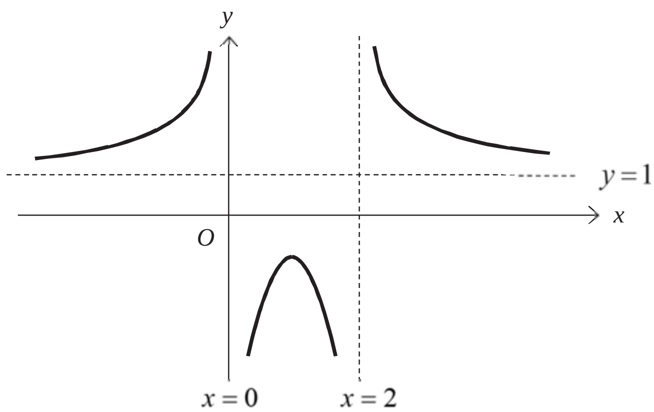
$$z^3 + kz^2 + \left(8 + 2\sqrt{2}\,\text{i}\right)z + 8\,\text{i} = 0,$$

where k is a constant to be determined, find the other roots, leaving your answers in exact cartesian form $x + y\text{i}$, showing your working. [3]

(ii) Hence solve the equation $\text{i}z^3 + kz^2 + \left(2\sqrt{2} - 8\,\text{i}\right)z - 8\,\text{i} = 0$, leaving your answers in exact cartesian form. [2]

(iii) Let z_0 be the root in **(i)** such that $\arg(z_0) > 0$. Find the smallest positive integer value of n such that $(\text{i}z_0)^n$ is a purely imaginary number. [2]

9 **(a)** The diagram below shows the graph of $y = \dfrac{1}{\text{f}(x)}$ with asymptotes $x = 0$, $x = 2$, and $y = 1$, and turning point $(1, -2)$.



(i) Given that $\text{f}(0) = \text{f}(2) = 0$, sketch the graph of $y = \text{f}(x)$, stating clearly the coordinates of any turning points and points of intersection with the axes, and the equations of any asymptotes. [3]

(ii) The function f is now defined for $x > k$ such that f^{-1} exists.
State the smallest value of k. On the same diagram, sketch the graphs of $y = \text{f}(x)$ and $y = \text{f}^{-1}(x)$, showing clearly the geometrical relationship between the two graphs. [3]

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(b) The function g is defined for $x > 0$ as

$$\mathrm{g} : x \mapsto 2^n x - 1, \quad \frac{1}{2^n} \le x < \frac{1}{2^{n-1}}, \text{ where } n \in \mathbb{N}.$$

(i) Fill in the blanks.

$$\mathrm{g}(x) = \begin{cases} \boxed{\qquad\qquad\qquad\qquad}, & \frac{1}{4} \le x < \frac{1}{2} \\ \boxed{\qquad\qquad\qquad\qquad}, & \frac{1}{2} \le x < 1 \end{cases}$$

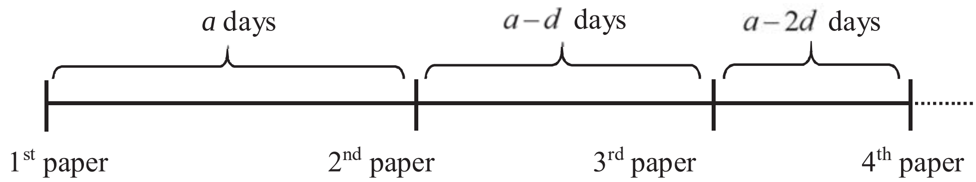
Hence sketch the graph $y = \mathrm{g}(x)$ for $\frac{1}{4} \le x < 1$. [3]

(ii) Show that $\mathrm{g}(x) = \mathrm{g}\left(\frac{x}{2}\right)$. [2]

(iii) Find the number of solutions of $\mathrm{g}(x) = x$ for $0.001 < x < 1$. [2]

10 David is preparing for an upcoming examination with 9 practice papers to complete in 90 days. The examination is on the 91st day. He is planning to spread out the practice papers according to the following criteria, and illustrated in the diagram below.

- He only completes 1 practice paper a day.
- He attempts the first practice paper on the first day.
- The duration between the first and the second practice paper is *a* days.
- The duration between each subsequent paper decreases by *d* days.
- He completes the last practice paper as close to the examination date as possible.



(i) By first writing down two inequalities in terms of *a* and *d*, determine the values of *a* and *d*. [4]

The mark for his *n*-th practice paper, u_n, can be modelled by the formula

$$u_n = 92 - 65(b)^n \text{ where } 0 < b < 1.$$

(ii) What is the significance of the number 92 in the formula? [1]

(iii) Find *m*, his average mark, for the nine practice papers he completed, leaving your answer in terms of *b*. [3]

(iv) Given that he scored higher than *m* from his fourth practice paper onwards, find the range of values of *b*. [2]

11 A toy paratrooper is dropped from a building and the attached parachute opens the moment it is released. The toy drops vertically and the distance it drops after t seconds is x metres. The motion of the toy can be modelled by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+k\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2=10,$$

where k is a constant.

By substituting velocity, $v=\frac{\mathrm{d}x}{\mathrm{d}t}$, write down a differential equation in v and t. [1]

Given that $\frac{\mathrm{d}v}{\mathrm{d}t}=6$ when $v=\sqrt{10}$, and that the initial velocity of the toy is zero, show that

$$v=\frac{5(1-\mathrm{e}^{-4t})}{1+\mathrm{e}^{-4t}},$$

and deduce the velocity of the toy in the long run. [6]

The toy is released from a height of 10 metres. Find the time it takes for the toy to reach the ground. [5]

12 In air traffic control, coordinates (x, y, z) are used to pinpoint the location of an aircraft in the sky within certain air space boundaries. In a particular airfield, the base of the control tower is at $(0,0,0)$ on the ground, which is the x-y plane. Assuming that the aircrafts fly in straight lines, two aircrafts, F_1 and F_2, fly along paths with equations

$$\mathbf{r}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(2\mathbf{i}-4\mathbf{j}+\mathbf{k}) \text{ and } x+2=\frac{y-1}{m}=\frac{3-z}{7}$$

respectively.

(i) What can be said about the value of m if the paths of the two aircrafts do not intersect? [3]

(ii) The signal detecting the aircrafts is the strongest when an aircraft is closest to the controller, who is in the control tower 3 units above the base. Find the distance of F_1 to the controller when the signal detecting it is the strongest. [3]

In a choreographed flying formation, the aircraft F_3 takes off from the point $(1,1,0)$ and flies in the direction parallel to $\mathbf{i}-\mathbf{k}$. The path taken by another aircraft, F_4, is the reflection of the path taken by F_2 along the path taken by F_3.

For the case when $m=5$, find

(iii) the cartesian equation of the plane containing all three flight paths. [2]

(iv) the vector equation of the line that describes the path taken by F_4. [4]

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