NAME: _______________________________ (   )   **CLASS: 4** (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020

S4

ADDITIONAL MATHEMATICS
4047/02

Paper 2

28 August 2020 Friday

2 hours 30 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the space at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	Marks	Question	Marks	Table of Penalties	
1		7			
2		8		Units	
3		9		Presentation	
4		10		Accuracy	
5		11		Total:	
6		12			
Parent's Name & Signature: Date:				100	

This document consists of **18** printed pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

1 **(a)** The equation of a curve is $y = mx^2 + (m-3)x + m$, where m is a constant.

Find the range of values of m for which the curve lies completely above the x-axis. [5]

(b) Given that $y = ax^2 - 4x + c$ is always negative, give an example of values of a and c which satisfy the condition. [2]

2 **(a)** Given that $2x^4 + 3x^3 + ax^2 - 9x + 9 = \left(x^2 - 1\right)\left(x - 2\right)\mathrm{Q}\left(x\right) - 3x^2 + bx + c$ is an identity, state, with reason, the degree of $\mathrm{Q}\left(x\right)$. [1]

(b) Find the value of *a*, of *b* and of *c*. [5]

(c) Hence, find the remainder when $2x^4+3x^3+ax^2-9x+9$ is divided by $(3x-1)$. [1]

3 **(a)** Given that $p=3^x$ and $q=3^y$, express $\log_3 \frac{p^7q}{243}$ in terms of x and y. [4]

(b) Given that $\log_2 x-\log_x x^2=\frac{1}{3}-\log_8 2x$, find the value of x, leaving your answer in index form. [4]

4 **(a)** Without using a calculator, express $\dfrac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}}$ in the form of $a\sqrt{10}+b\sqrt{3}$. [4]

(b) Without the use of a calculator, solve the equation $\sqrt[3]{27^{x}}-81^{x+1}=0$. [3]

5 **(a)** **(i)** Given the curve $y = -\left|3x - x^2\right| + 4$, find the x-coordinates of the points where the curve meets the x-axis. [2]

(ii) Sketch the curve $y = -\left|3x - x^2\right| + 4$, giving the coordinates of the turning point and of the points where the curve meets the axes. [3]

(b) Explain why there are only two solutions to the equation $-\left|3x - x^2\right| = k - 4$ for $k < 1.75$. [2]

(c) Determine the maximum value for m for which the line $y = mx + 1$ intersects the graph of $y = -\left|3x - x^2\right| + 4$ at three points. [1]

6 **(i)** Express $\dfrac{7x+11}{(x-1)(x+2)^2}$ in partial fractions. [4]

(ii) Hence, find $\displaystyle\int \frac{7x+11}{2(x-1)(x+2)^2}\,dx$. [3]

7 A piece of wire which has a fixed length of k cm long is bent to form a rectangle. Show that the area of the rectangle is a maximum when it is a square. [5]

8 Given a circle with the equation $(2x+5)(x+2)+(2y+1)(y-5)=0$,

(i) Express the equation of the circle in standard form. [5]

(ii) Find the length of the chord when the line $y = -2x$ cuts the circle. [5]

9 **(a)** **(i)** Prove the identity $\sin x \cos x + \cot x \cos^2 x = \cot x$. [4]

(ii) Hence, solve $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1$ for $0 \leq x \leq \pi$. [3]

(b) **(i)** On the same axes, sketch the graphs of $y = 3\sin x - 1$ and $y = \tan\frac{x}{2}$ for $0 \le x \le 2\pi$. [5]

(ii) Hence, state the number of solutions of $3\sin x - 1 = \tan\frac{x}{2}$ for $0 \le x \le 2\pi$. [1]

10 Two particles, A and B, leave a point O at the same time and travel in the same direction along the same straight line.
Particle A starts with a velocity of 9 m/s and moves with a constant acceleration of 2 m/s^2.
Particle B starts from rest and moves with an acceleration of a m/s^2, where $a = 1 + \frac{t}{3}$ and t seconds is the time since leaving O. Find

(a) an expression for the velocity of each particle in terms of t, [3]

(b) an expression for the displacement of each particle in terms of t, [3]

(c) the distance from O at which particle B collides with A, [3]

(d) the speed of each particle at the point of collision. [2]

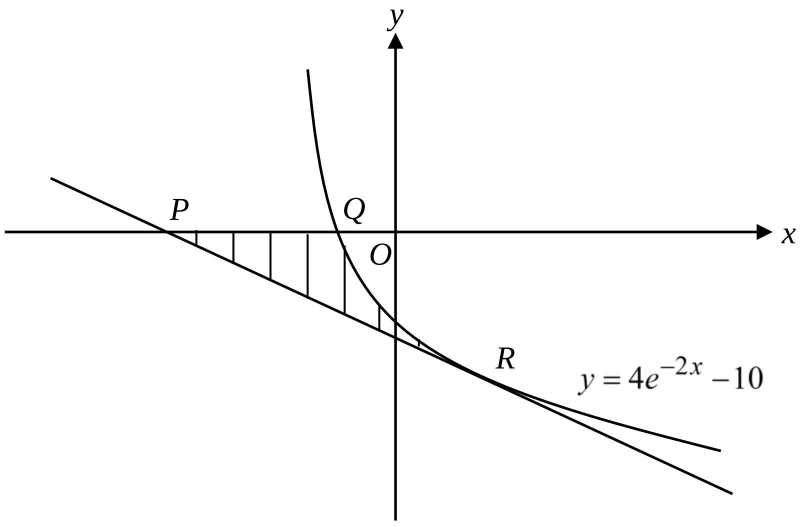
11 Given that $\cos A = p$ and that A is acute, express the following in terms of p.

(i) $\sin 2A$ [3]

(ii) $\cos \frac{A}{2}$ [3]

12 The diagram shows the curve, $y = 4e^{-2x} - 10$. The curve crosses the x-axis at Q. The line PR is a tangent to the curve at R and intersects the x-axis at P. The x-coordinate of R is ln 2.

Find the area of the shaded region, PQR, which is the region enclosed by curve, the x-axis and the line PR correct to 3 significant figures. [11]



Continuation of working space for Question 12.

- END OF PAPER –

Answer key for AM paper 2

1

(a) $m > 1$

(b) $a = -1$ and $c = -5$ or
$a = -2$ and $c = -2$ or
$a = -5$ and $c = -1$ or any pairs of values that fulfill the above criteria

2

(a) Since degree of dividend = degree of quotient + degree of divisor,
degree of $\mathrm{Q}(x) = 1$

ALTERNATIVE: Must multiply with x^3 to give degree 4 in the polynomial on the left.

(b) $a = -19, \quad b = -6, \quad c = -5$ (c) $\frac{326}{81}$

3

(a) $7x + y - 5$

(b) $x = 2^{\frac{3}{2}}$ or $x = 8^{\frac{1}{2}}$ or $x = 64^{\frac{1}{4}}$

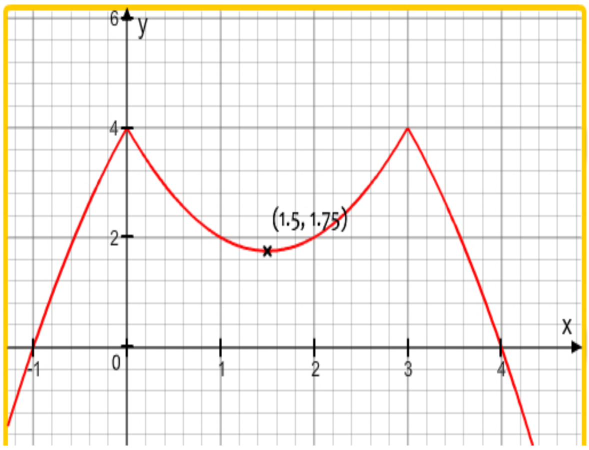
4

(a) $\frac{4}{13}\sqrt{10} - \frac{7}{13}\sqrt{3}$ (b) $x = -\frac{4}{3}$

5

(a)(i) $x = 4$ or $x = -1$

(ii)



(b)

$-\left|3x - x^2\right| = k - 4$

$-\left|3x - x^2\right| + 4 = k$

$y = k$

$y = k$ is a **horizontal line** and when k is lesser than 2.25, it will be **below the turning point** and so it will only **intersect the curve at the two outer arms thereby giving two solutions only.**

(c) $m = 1$

6

(i) $\frac{2}{x-1} - \frac{2}{x+2} + \frac{1}{(x+2)^2}$

(ii) $\ln(x-1) - \ln(x+2) - \frac{1}{2(x+2)} + c$

7

$x = \frac{k}{4}$, $\frac{d^2A}{dx^2} = -2 < 0$

Therefore, since the stationary value occurs when the sides of the rectangle are $\frac{k}{4}$ cm, and it is a maximum value, the maximum area of the rectangle occurs when it is a square.

8

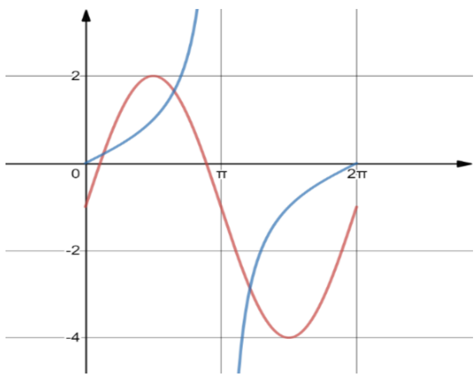
(i) $\left(x + \frac{9}{4}\right)^2 + \left(y - \frac{9}{4}\right)^2 = \frac{61}{8}$

(ii) 5.14 units (to 3 s.f.)

9

(a) (ii) $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$ rad

(b)(i)



(ii) From the sketch, the two functions intersect at three points. Hence there are three solutions for the equation for $0 \le x \le 2\pi$

10

(a) $v_B = t + \frac{1}{6}t^2$

(b) $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3$

(c) 486 m

(d) 72 m/s

11	(i) $2p\sqrt{1-p^2}$ (ii) $\sqrt{\dfrac{p+1}{2}}$	12	$R(\ln 2,-9)$ $P\left(\ln 2-\dfrac{9}{2},0\right)$ $Q\left(-\dfrac{1}{2}\ln\dfrac{5}{2},0\right)$ Area = 13.2 units^2