TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME: ____

CIVICS GROUP: ____

H2 MATHEMATICS

9758/02

Paper 2

25 SEPTEMBER 2019

3 hours

Candidates answer on the question paper.

Additional material: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

For Examiners' Use	
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Total	

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

[Turn Over

Section A: Pure Mathematics [40 marks]

1 Given that $y=\sqrt{\left(5-\mathrm{e}^{2x}\right)}$, show that

$$y\frac{\mathrm{d}y}{\mathrm{d}x}=-\mathrm{e}^{2x} \ . \qquad [1]$$

By further differentiation of this result, find the Maclaurin series for y up to and including the term in x^2. [4]

2 Let z_1 and z_2 be the roots of the equation

$$z^2-\mathrm{i}kz-1=0$$

where $0<k<2$ and $0<\arg\left(z_1\right)<\arg\left(z_2\right)<\pi$.

(i) Find z_1 and z_2 in cartesian form, $x+\mathrm{i}y$, where x and y are real constants in terms of k. [3]

For the rest of the question, let $\arg\left(z_1\right)=\theta$, $0<\theta<\frac{\pi}{2}$.

Let w be a complex number such that wz_1 is purely imaginary and $\arg\left(\frac{z_1}{w}\right)=\frac{7\pi}{6}$.

(ii) Show that $\arg\left(z_1\right)=\frac{\pi}{3}$. [3]

(iii) Find z_2, leaving your answer in the exact form. [3]

3 The plane p_1 contains the point A with coordinates $(1,\ 2,\ 8)$ and the line l with equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix},$$ where λ is a real parameter.

(i) Show that a cartesian equation of plane p_1 is $3x - y + z = 9$. [2]

The foot of perpendicular from point A to line l is denoted as point F.

(ii) Find the coordinates of point F. [3]

(iii) Point B has coordinates $(1, -4,\ 2)$. Find the exact area of triangle ABF. [2]

(iv) Point C has coordinates $(-1,\ 6,\ 6)$. By finding the shortest distance from point C to p_1, find the exact volume of tetrahedron $ABFC$. [4]

[Volume of tetrahedron = $\frac{1}{3} \times$ base area $\times$ perpendicular height]

(v) Point D lies on the line segment AC such that $AD : DC = 1 : 3$. Another plane p_2 is parallel to p_1 and contains point D. Find a cartesian equation of p_2. [2]

4 A designer is tasked to design a 3-dimensional ornament for the company. He then programs two curves, C_1 and C_2, into the computer software. The curve C_1 has the equation $y=\sqrt{\left(2-x^2\right)}$ and the curve C_2 has the equation $y=x^3$. The coordinates of the point of intersection of C_1 and C_2 is $(1,1)$.

(i) Find the exact area of the finite region bounded by C_1, C_2 and the positive x-axis. [You may use the substitution $x=\sqrt{2}\sin\theta$ where $0\le\theta\le\frac{\pi}{2}$.] [6]

(ii) The designer wants to know how much material is needed to construct the 3-dimensional ornament. He finds out that the surface area generated by the segment of a curve $y=\mathrm{f}(x)$ between $x=a$ and $x=b$ rotated through 360° about the x-axis is given by

$$S=\int_a^b 2\pi y\sqrt{\left(1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\,\mathrm{d}x \text{ where } y=\mathrm{f}(x),\ y\ge 0,\ a\le x\le b.$$

The 3-dimensional ornament is formed when the finite region bounded by C_1, C_2 and the positive x-axis is rotated through 360° about the x-axis. Find the exact surface area of the 3-dimensional ornament. [7]

Section B: Probability and Statistics [60 marks]

5 A biased die in the form of a regular tetrahedron has its four faces labelled 2, 3, 4 and 5, with one number on each face. The die is tossed and X is the random variable denoting the number on the face which the die lands. The probability distribution of X is shown in the table below, where $0<v<u<1$.

x	2	3	4	5
$\mathrm{P}(X=x)$	u	v	u	v

(i) Find $\mathrm{E}(X)$ in terms of u. [2]

(ii) Given that $\mathrm{Var}(X)=1.16$, find u and v. [4]

6 A computer game consists of at most 3 rounds. The game will stop when a player clears 2 rounds or does not clear 2 consecutive rounds. The probability that a player clears round 1 is 0.6. The conditional probability that the player clears round 2 given that he clears round 1 is half the probability that he clears round 1. The conditional probability that the player clears round 2 given that he does not clear round 1 is the same as the probability that he clears round 1.

(i) Find the probability that a player plays 3 rounds. [1]

(ii) Find the probability that a player clears round 1 given that he does not clear round 2. [2]

(iii) The total probability that a player plays 3 rounds and clears round 3 is 0.2. Find the probability that a player clears exactly 2 rounds. [2]

In order to play the computer game, Eric needs to type a 6-digit passcode to unlock the game. The 6-digit passcode consists of digits 0-9 and the digits do not repeat.

How many possible passcodes can there be if

(iv) the 6-digit passcode is odd? [2]

(v) there are exactly 3 odd digits in the 6-digit passcode? [2]

7 MHL bakery sells mini breads that weighs an average of 45 grams each. A customer claims that the bakery is overstating the average weight of mini breads. To test this claim, a random sample of 80 mini breads are selected from MHL bakery and the weight, x grams, of each mini bread is measured. The results are summarised as follows.

$$n = 80 \qquad \sum x = 3571 \qquad \sum x^2 = 159701$$

Calculate unbiased estimates of the population mean and variance of the weight of mini breads. [2]

Test, at the 4% level of significance, whether there is sufficient evidence to support the customer's claim. [4]

From past records, it is known that the weights of the mini breads from MHL bakery are normally distributed with standard deviation 1.5 grams. To further investigate the customer's claim, the bakery records the weights of another 20 randomly selected mini breads and the average weight for the second sample is k grams.

Based on the combined sample of 100 mini breads, find the range of values of k such that the customer's claim is valid at the 4% level of significance. [4]

8 In a chemical reaction, the amount of catalyst used, x grams, and the resulting reaction times, y seconds, were recorded and the results are given in the table.

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
y	62.1	51.2	44.1	39.1	35.0	k	33.0	31.4	29.5

The equation of the regression line of y on x is $y = 68.8067 - 7.12667x$, correct to 6 significant figures.

(i) Show that $k = 37.3$, correct to 1 decimal place. [2]

(ii) Draw a scatter diagram for these values, labelling the axes clearly. [1]

It is suggested that the relationship between x and y can be modelled by one of the following formulae

(A) $y = a + bx$

(B) $y = c + dx^2$

(C) $y = e + \frac{f}{x}$

where a, b, c, d, e and f are constants.

(iii) Find the value of the product moment correlation coefficient for each model. Explain which is the best model and find the equation of a suitable regression line for this model. [5]

(iv) By using the equation of the regression line found in part **(iii)**, estimate the reaction time when the amount of catalyst used is 4.2 grams. Comment on the reliability of your estimate. [2]

9 The masses in grams of Envy apples have the distribution $\mathrm{N}\left(\mu, \sigma^2\right)$.

(i) For a random sample of 8 Envy apples, it is given that the probability that the sample mean mass is less than 370 grams is 0.25, and the probability that the total mass of these 8 Envy apples exceeds 3000 grams is 0.5. Find the values of μ and σ. [4]

For the rest of the question, use $\mu = 380$ and $\sigma = 20$.

(ii) Find the probability that the total mass of 8 randomly chosen Envy apples is between 2900 grams and 3100 grams. [2]

The masses in grams of Bravo apples have the distribution $\mathrm{N}\left(250, 18^2\right)$.

To make a fruit platter, a machine is used to slice the apples and remove the cores. After slicing and removing the cores, the mass of an Envy apple and the mass of a Bravo apple will be reduced by 30% and 20% respectively. A fruit platter consists of 8 randomly chosen Envy apples and 12 randomly chosen Bravo apples.

(iii) Find the probability that the total mass of fruits, after slicing and removing the cores, in a fruit platter exceeds 4.5 kg. [4]

(iv) State an assumption needed for your calculations in parts **(ii)** and **(iii)**. [1]

To beautify the fruit platter, fruit carving is done on the apples after slicing and removing their cores. The carving reduced the masses of each apple (after slicing and removing its core) by a further 10%.

Let *p* be the probability that the total mass of fruits in a fruit platter, with carving done, exceeds 4.1 kg. Without calculating *p*, explain whether *p* is higher, lower or the same as the answer in part **(iii)**. [1]

10 **(a)** In a packet of 10 sweets, it is given that six of them are red, three of them are yellow and the remaining one is blue. 5 sweets are chosen randomly from the packet of sweets and R denotes the number of sweets that are red. Explain clearly why R cannot be modelled by a binomial distribution. [1]

(b) In a food company, a large number of sweets are produced daily and it is given that $100p\%$ of the sweets produced are red. The sweets are packed into packets of 10 each. Assume now that the number of sweets that are red in a packet follows a binomial distribution.

(i) It is given that the probability of containing exactly five red sweets in a randomly chosen packet of sweets is 0.21253. Show that p satisfies an equation of the form $p(1-p)=k$, where k is a constant to be determined. Hence find the possible values of p. [3]

For the rest of the question, use $p=0.6$.

(ii) A packet of sweets is randomly chosen. Find the probability that there is at most 8 red sweets given that there is more than 2 red sweets. [3]

(iii) Two packets of sweets are selected at random. Find the probability that one of the packets contains at most 5 red sweets and the other packet contains at least 5 red sweets. [3]

(iv) Two packets of sweets are selected at random. Find the probability that the difference in the number of red sweets in the two packets is at least 8. [3]

End of Paper