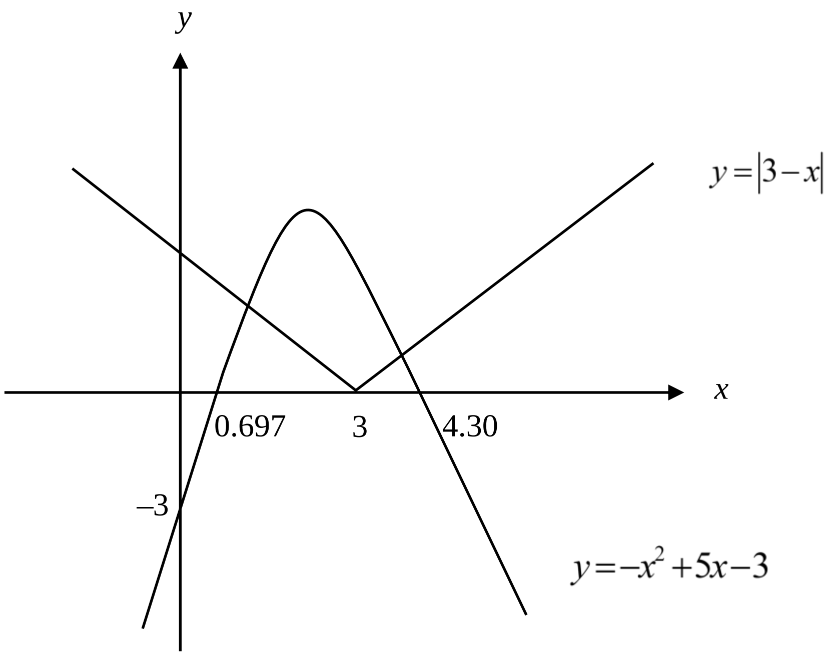
Qn	Solution
1	**Sequences and Series**
(i)	$\frac{3}{r}+\frac{2}{r+1}-\frac{5}{r+2}=\frac{3(r+1)(r+2)+2r(r+2)-5r(r+1)}{r(r+1)(r+2)}$ $=\frac{3r^2+9r+6+2r^2+4r-5r^2-5r}{r(r+1)(r+2)}$ $=\frac{8r+6}{r(r+1)(r+2)}$
(ii)	$\sum_{r=1}^{n}\frac{4r+3}{r(r+1)(r+2)}=\frac{1}{2}\sum_{r=1}^{n}\frac{8r+6}{r(r+1)(r+2)}$ $=\frac{1}{2}\sum_{r=1}^{n}\left(\frac{3}{r}+\frac{2}{r+1}-\frac{5}{r+2}\right)$ $=\frac{1}{2}\left(\frac{3}{1}+\frac{2}{2}-\frac{5}{3}\right.$ $+\frac{3}{2}+\frac{2}{3}-\frac{5}{4}$ $+\frac{3}{3}+\frac{2}{4}-\frac{5}{5}$ $+\frac{3}{4}+\frac{2}{5}-\frac{5}{6}$ $+$ $\vdots$ $+\frac{3}{n-2}+\frac{2}{n-1}-\frac{5}{n}$ $+\frac{3}{n-1}+\frac{2}{n}-\frac{5}{n+1}$ $\left.+\frac{3}{n}+\frac{2}{n+1}-\frac{5}{n+2}\right)$ $=\frac{1}{2}\left(\frac{3}{1}+\frac{2}{2}+\frac{3}{2}-\frac{5}{n+1}+\frac{2}{n+1}-\frac{5}{n+2}\right)$ $=\frac{11}{4}-\frac{3}{2(n+1)}-\frac{5}{2(n+2)}$
	$\sum_{r=3}^{n}\frac{4r-5}{r(r-1)(r-2)}=\sum_{r=1}^{n-2}\frac{4r+3}{r(r+1)(r+2)}$ $=\frac{11}{4}-\frac{3}{2(n-1)}-\frac{5}{2n}$

Qn	Solution
2	**Integration**
(a)	$\int \frac{1}{\sqrt{\left(1-x^2\right)\sin^{-1} x}}\ \mathrm{d}x = \int \frac{1}{\sqrt{\left(1-x^2\right)}}\left(\sin^{-1} x\right)^{-\frac{1}{2}}\ \mathrm{d}x$ $= \frac{\left(\sin^{-1} x\right)^{\frac{1}{2}}}{\frac{1}{2}} + C$ $= 2\sqrt{\sin^{-1} x} + C$
(b)	$\int \frac{x-3}{x^2-2x+4}\ \mathrm{d}x = \frac{1}{2}\int \frac{2x-2}{x^2-2x+4}\ \mathrm{d}x - \int \frac{2}{\left(x-1\right)^2+3}\ \mathrm{d}x$ $= \frac{1}{2}\ln\left(x^2-2x+4\right) - \frac{2}{\sqrt{3}}\tan^{-1}\frac{x-1}{\sqrt{3}} + C$ **Note:** $x^2-2x+4=\left(x-1\right)^2+3>0$ **.** Therefore, modulus is not required for ln(.)

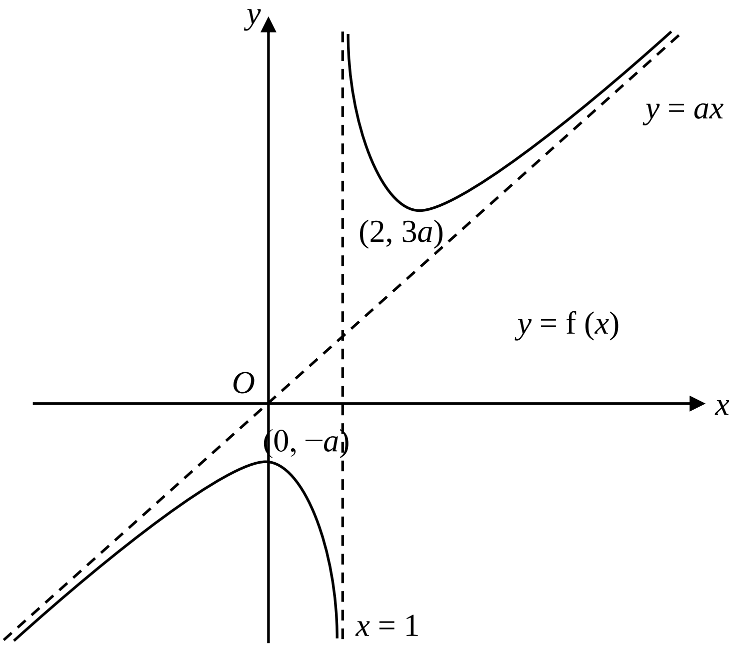
Qn	Solution
3	**Chapter 11 Definite Integral**
	$\int_0^8 y\ \mathrm{d}x$ $= \int_0^{\frac{2\sqrt{2}}{a}} \mathrm{e}^{at}\left(2a^2 t\right)\ \mathrm{d}t$ $= 2a^2\left\{\left[\frac{\mathrm{e}^{at}}{a}(t)\right]_0^{\frac{2\sqrt{2}}{a}} - \int_0^{\frac{2\sqrt{2}}{a}} \frac{\mathrm{e}^{at}}{a}(1)\ \mathrm{d}t\right\}$ $= 2a^2\left\{\left[\frac{\mathrm{e}^{2\sqrt{2}}}{a}\left(\frac{2\sqrt{2}}{a}\right)-0\right]-\left[\frac{\mathrm{e}^{at}}{a^2}\right]_0^{\frac{2\sqrt{2}}{a}}\right\}$ $= 2a^2\left[\frac{\mathrm{e}^{2\sqrt{2}}2\sqrt{2}}{a^2}-\left(\frac{\mathrm{e}^{2\sqrt{2}}}{a^2}-\frac{1}{a^2}\right)\right]$ $= 2\mathrm{e}^{2\sqrt{2}}\left(2\sqrt{2}-1\right)+2$ when $x=8, \left(at\right)^2=8 \Rightarrow t=\frac{2\sqrt{2}}{a}$ when $x=0, \left(at\right)^2=0 \Rightarrow t=0$ $x=a^2t^2$ $\frac{\mathrm{d}x}{\mathrm{d}t}=2a^2t$

Qn	Solution
4	**Graphing and Inequalities**
	
	$-x^2+5x-3=\|3-x\|$ $-x^2+5x-3=3-x$ or $-x^2+5x-3=-(3-x)$ For $x<3$: $-x^2+5x-3=3-x$ $x^2-6x+6=0$ $x=\dfrac{6\pm\sqrt{6^2-4(1)(6)}}{2}$ $=\dfrac{6\pm\sqrt{12}}{2}$ $=3\pm\sqrt{3}$ $=3-\sqrt{3}$ since $x<3$ For $x>4$: $-x^2+5x-3=-(3-x)$ $x^2-4x=0$ $x(x-4)=0$ $x=0$ (reject since $x>3$) or $x=4$ $-x^2+5x-3<\|3-x\|$ Using graph, $x<3-\sqrt{3}$ or $x>4$

Qn	Solution
5	**Maclaurin Series**
	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=k\cos kx$ $\frac{\mathrm{d}y}{\mathrm{d}x}=ky\cos kx$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=ky\left(-k\sin kx\right)+k\frac{\mathrm{d}y}{\mathrm{d}x}\cos kx$ $\quad=-k^2y\sin kx+\frac{\mathrm{d}y}{\mathrm{d}x}\left(k\cos kx\right)$ $\quad=-k^2y\ln y+\frac{\mathrm{d}y}{\mathrm{d}x}\left(\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ $\quad\therefore\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+k^2y\ln y-\frac{1}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=0$ When $x=0, y=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=k$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=k^2$ $y=1+kx+\frac{k^2x^2}{2}+\ldots$
	$y=\mathrm{e}^{\sin kx}$ $\quad=\mathrm{e}^{\left(kx-\frac{(kx)^3}{3!}+\ldots\right)}$ $\quad=1+\left(kx-\frac{k^3x^3}{6}\right)+\frac{\left(kx-\frac{k^3x^3}{6}\right)^2}{2}+\frac{\left(kx-\frac{k^3x^3}{6}\right)^3}{6}+\ldots$ $\quad=1+kx-\frac{k^3x^3}{6}+\frac{1}{2}(k^2x^2)+\frac{1}{6}(k^3x^3)+\ldots$ $\quad=1+kx+\frac{k^2x^2}{2}+\ldots$ (verified) Coefficient of $x^3=0$

Qn	Solution
6	**Applications of Differentiation**
(a)	The water forms a smaller cone of radius r and height h. From the diagram, by similar triangles, $\frac{r}{h}=\frac{4}{8}$ $r=\frac{h}{2}$ Let the volume of water in the cone be V. $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $=\frac{1}{3}\pi\left(\frac{h^2}{4}\right)h$ $=\frac{\pi}{12}h^3$ $V=\frac{\pi}{12}h^3$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\pi}{4}h^2$ Given: $\frac{\mathrm{d}V}{\mathrm{d}t}=-1.5\text{ cm}^3/\text{s}$ When $h=2$, $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ $-1.5=\frac{\pi}{4}(2)^2\times\frac{\mathrm{d}h}{\mathrm{d}t}$ $\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{3}{2\pi}$ cm/s The rate at which the water level is **decreasing** at $\frac{3}{2\pi}$ cm/s.
(b)	$\tan\theta=\frac{DM}{5}$ $\quad\therefore DM=5\tan\theta$ $\cos\theta=\frac{5}{AD}$ $\quad\therefore AD=5\sec\theta$ $DA+AB+BC=20$ $5\sec\theta+AB+5\sec\theta=20$ $\therefore AB=20-10\sec\theta$ Area of trapezium *ABCD* $=\frac{1}{2}(5)(AB+DC)$ $=\frac{5}{2}\left(2(20-10\sec\theta)+2(5\tan\theta)\right)$ $=100-50\sec\theta+25\tan\theta$ (shown)

(ii)	$A = 100 - 50\sec\theta + 25\tan\theta$ $\frac{dA}{d\theta} = -50\sec\theta\tan\theta + 25\sec^2\theta = 0$ $25\sec\theta(-2\tan\theta + \sec\theta) = 0$ $25\sec\theta = 0$ or $2\tan\theta = \sec\theta$ (NA) $\sin\theta = \frac{1}{2}$ $\therefore \theta = \frac{\pi}{6}$ $A = 100 - 50\sec\left(\frac{\pi}{6}\right) + 25\tan\left(\frac{\pi}{6}\right)$ $= 100 - 50\left(\frac{2}{\sqrt{3}}\right) + 25\left(\frac{1}{\sqrt{3}}\right)$ $= 100 - \frac{75}{\sqrt{3}}$ $= \left(100 - 25\sqrt{3}\right)\text{cm}^2$

Qn	Solution
7	**Functions and Transformation of curves**
(a) (i)	 Let $y = a\left(x + \frac{1}{x-1}\right)$ $\frac{dy}{dx} = 0 \Rightarrow x = 0 \text{ or } 2$
(a) (ii)	Since the line $y = 4a$ cuts the graph of $y = f(x)$ more than once, f is not a one-one function, hence f does not have an inverse.
(a) (iii)	$R_f = (-\infty, -a] \cup [3a, \infty)$ $D_g = \mathbb{R} \setminus \{-1, 1\}$ Since $a > 1$, $-a < -1$ and $3a > 1$ Since $R_f \subseteq D_g$, $\therefore$gf exists.
(b)	$y = e^{\frac{x^2-4}{12}}$ $\downarrow$ **A**: Replace x by $\frac{x}{\left(\frac{1}{2}\right)} = 2x$ $y = e^{\frac{(2x)^2-4}{12}} = e^{\frac{x^2-1}{3}}$ $\downarrow$ **B**: Replace y by $y+1$ $y+1 = e^{\frac{x^2-1}{3}}$ $\downarrow$ **C**: Replace y by x and x by y $x+1 = e^{\frac{y^2-1}{3}}$ $\frac{y^2-1}{3} = \ln(x+1)$ $y^2 = 3\ln(x+1)+1$ $\because y > 0, y = \sqrt{3\ln(x+1)+1}$

	$\therefore y = \mathrm{q}(x) = \sqrt{3\ln(x+1)+1}$ Range of C: $\left(\mathrm{e}^{-\frac{1}{3}}, \infty\right)$ $\left(\mathrm{e}^{-\frac{1}{3}}, \infty\right) \xrightarrow{\mathbf{A}} \left(\mathrm{e}^{-\frac{1}{3}}, \infty\right) \xrightarrow{\mathbf{B}} \left(\mathrm{e}^{-\frac{1}{3}} - 1, \infty\right)$ $\therefore \mathrm{D_q} = \left(\mathrm{e}^{-\frac{1}{3}} - 1, \infty\right)$

Qn	Solution
8	
(i)	Equation of l_1 : $\mathbf{r} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$ where $\lambda \in \mathbb{R}$ Equation of p_1 : $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 3$ Normal of p_2 = $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix} = 2\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$ Hence, equation of p_2 : $\mathbf{r} \cdot \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} = 2 - 4 + 3 = 1$ Cartesian equation of p_2 : $-x + 4y + z = 1$
(ii)	Solving $x + z = 3 ---(1)$ $-x + 4y + z = 1 ---(2)$ From GC: $x = 3 - \mu$, $y = 1 - \frac{1}{2}\mu$, $z = \mu$ Hence, equation of l_2 : $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$ where $\alpha \in \mathbb{R}$
(iii)	Let F be the foot of perpendicular from B to p_2 **Method 1:** Let l_{BF} : $r = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} + t\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$ where $t \in \mathbb{R}$ Sub into equation of p_2 : $\begin{pmatrix} -t \\ 4+4t \\ 3+t \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} = 1$ $t + 16 + 16t + 3 + t = 1$ $t = -1$ Hence, $\overrightarrow{OF} = \begin{pmatrix} 0-(-1) \\ 4-4 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ Perpendicular distance from B to p_2 $= k = \left\|\overrightarrow{BF}\right\| = \left\|\begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}\right\| = 3\sqrt{2}$

	Method 2: Since F is on l_2 as p_1 and p_2 are perpendicular planes, $\overrightarrow{OF} =$ $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$ $\overrightarrow{BF} = \overrightarrow{OF} - \overrightarrow{OB} = \begin{pmatrix} 3-2\alpha \\ 1-\alpha \\ 2\alpha \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 3-2\alpha \\ -3-\alpha \\ -3+2\alpha \end{pmatrix}$ $\overrightarrow{BF} \cdot \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} = 0$ $\begin{pmatrix} 3-2\alpha \\ -3-\alpha \\ -3+2\alpha \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} = 0$ $-6+4\alpha+3+\alpha-6+4\alpha = 0$ $\alpha = 1$ $\overrightarrow{OF} = \begin{pmatrix} 3-(1) \\ 1-1 \\ 2(1) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ Perpendicular distance from B to p_2 $= k = \left\|\overrightarrow{BF}\right\| = \left\|\begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}\right\| = 3\sqrt{2}$
(iv)	Since the perpendicular distance from B to l_2 is $3\sqrt{2}$, the lines on p_1 with a distance of $3\sqrt{2}$ must be parallel to l_2. Using ratio theorem, find the reflected point B' in the line l_2 $\overrightarrow{OB'} = 2\overrightarrow{OF} - \overrightarrow{OB} = 2\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$ Hence, equation of lines: $\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$ where $\beta \in \mathbb{R}$ or $\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$ $\gamma \in \mathbb{R}$

Qn	Solution
9	**Complex Numbers**
(a)	Given $z = x + \mathrm{i}y, \quad x, y \in \mathbb{R}$ $\dfrac{(z^2)^*}{z} = \dfrac{(z^*)^2}{z}$ $= \dfrac{(x - y\mathrm{i})^2}{x + y\mathrm{i}}$ $= \dfrac{(x - y\mathrm{i})^2 (x - y\mathrm{i})}{(x + y\mathrm{i})(x - y\mathrm{i})}$ $= \dfrac{(x - y\mathrm{i})^3}{x^2 - (y\mathrm{i})^2}$ $= \dfrac{x^3 - 3x^2(y\mathrm{i}) + 3x(y\mathrm{i})^2 - (y\mathrm{i})^3}{x^2 + y^2}$ $= \dfrac{x^3 - 3x^2y\mathrm{i} - 3xy^2 + y^3\mathrm{i}}{x^2 + y^2}$ $= \dfrac{x^3 - 3xy^2}{x^2 + y^2} + \dfrac{-3x^2y + y^3}{x^2 + y^2}\mathrm{i}$ Given $\dfrac{(z^2)^*}{z}$ is real $\Longrightarrow \mathrm{Im}\left[\dfrac{(z^2)^*}{z}\right] = 0$ $\Rightarrow \dfrac{-3x^2y + y^3}{x^2 + y^2} = 0$ $-3x^2y + y^3 = 0$ $y(y^2 - 3x^2) = 0$ $y(y - \sqrt{3}x)(y + \sqrt{3}x) = 0$ $y = 0$ (rejected as y is non-zero) , $\quad y = \sqrt{3}x \quad$ or $\quad y = -\sqrt{3}x$ Since $\lvert z \rvert = 1 \Rightarrow x^2 + y^2 = 1$, For $y = \pm\sqrt{3}x,$, $\quad x^2 + (\pm\sqrt{3}x)^2 = 1$ $x^2 = \dfrac{1}{4}$ $x = \pm\dfrac{1}{2} \quad \Rightarrow y = \pm\dfrac{\sqrt{3}}{2}$ $\therefore$ Possible values of z are $\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}\mathrm{i}, \quad -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}\mathrm{i}, \quad \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\mathrm{i}$ or $-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\mathrm{i}$.

(b)	Let $z = x + \mathrm{i}y, \qquad x, y \in \mathbb{R}$ $(x+\mathrm{i}y)^2 = 33 + 56\mathrm{i}$ $x^2 + 2xy\mathrm{i} - y^2 = 33 + 56\mathrm{i}$ Comparing real and imaginary parts, $x^2 - y^2 = 33$ --- (1) $2xy = 56$ $y = \dfrac{28}{x}$ --- (2) Substitute (2) into (1): $x^2 - \left(\dfrac{28}{x}\right)^2 = 33$ $x^4 - 33x^2 - 784 = 0$ $(x^2 - 49)(x^2 + 16) = 0$ Since x is real , $x = -7$ or $x = 7$ When $x = -7,\ y = -4$ $\quad \therefore z = -7 - 4\mathrm{i}$ When $x = 7,\ y = 4$ $\quad \therefore z = 7 + 4\mathrm{i}$ $\therefore$ the roots of the equation are $-7-4\mathrm{i}$ and $7+4\mathrm{i}$. $w^2 = -33 + 56\mathrm{i}$ $-w^2 = 33 - 56\mathrm{i}$ $(-w^2)^* = 33 + 56\mathrm{i}$ (conjugate both sides) $-(w^*)^2 = 33 + 56\mathrm{i}$ $\mathrm{i}^2 (w^*)^2 = 33 + 56\mathrm{i}$ $(\mathrm{i}w^*)^2 = 33 + 56\mathrm{i}$ Replace z by $\mathrm{i}w^*$, $\mathrm{i}w^* = -7 - 4\mathrm{i}$ or $\mathrm{i}w^* = 7 + 4\mathrm{i}$ $-w^* = -7\mathrm{i} + 4$ $\qquad -w^* = 7\mathrm{i} - 4$ $w^* = 7\mathrm{i} - 4$ $\qquad w^* = -7\mathrm{i} + 4$ $w = -4 - 7\mathrm{i}$ $\qquad w = 4 + 7\mathrm{i}$

Qn	Solution
10	**Differential Equations**
(i)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+0.1\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2=10$ Since $y=\frac{\mathrm{d}x}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $\therefore\frac{\mathrm{d}y}{\mathrm{d}t}+0.1y^2=10$ $\frac{\mathrm{d}y}{\mathrm{d}t}=10-0.1y^2$ (shown)
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}t}=10-0.1y^2=0.1\left(100-y^2\right)$ $\frac{1}{100-y^2}\frac{\mathrm{d}y}{\mathrm{d}t}=0.1$ $\int\frac{1}{10^2-y^2}\,\mathrm{d}y=\int0.1\,\mathrm{d}t$ $\frac{1}{2(10)}\ln\left\|\frac{10+y}{10-y}\right\|=0.1t+C$ $\ln\left\|\frac{10+y}{10-y}\right\|=2t+20C$ $\frac{10+y}{10-y}=\pm\mathrm{e}^{2t+20C}$ $\frac{10+y}{10-y}=A\mathrm{e}^{2t}$, where $A=\pm\mathrm{e}^{20C}$ When $t=0, y=0, \therefore A=1$ $10+y=\mathrm{e}^{2t}(10-y)$ $y\left(1+\mathrm{e}^{2t}\right)=10\left(\mathrm{e}^{2t}-1\right)$ $y=\frac{10\left(\mathrm{e}^{2t}-1\right)}{\mathrm{e}^{2t}+1}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{10\left(\mathrm{e}^{2t}-1\right)}{\mathrm{e}^{2t}+1}$ $=10\left(\frac{\mathrm{e}^{2t}}{\mathrm{e}^{2t}+1}-\frac{1}{\mathrm{e}^{2t}+1}\right)$ $=10\left(\frac{\mathrm{e}^{2t}}{\mathrm{e}^{2t}+1}-\frac{\mathrm{e}^{-2t}}{1+\mathrm{e}^{-2t}}\right)$ $x=5\int\left(\frac{2\mathrm{e}^{2t}}{\mathrm{e}^{2t}+1}-\frac{2\mathrm{e}^{-2t}}{1+\mathrm{e}^{-2t}}\right)\mathrm{d}t$ $=5\left(\ln\left\|\mathrm{e}^{2t}+1\right\|+\ln\left\|1+\mathrm{e}^{-2t}\right\|\right)+D$ $=5\left(\ln\left(\mathrm{e}^{2t}+1\right)+\ln\left(1+\mathrm{e}^{-2t}\right)\right)+D$, $\because\mathrm{e}^{2t}+1>0$ and $1+\mathrm{e}^{-2t}>0$ $=5\ln\left(2+\mathrm{e}^{2t}+\mathrm{e}^{-2t}\right)+d$

	When $t=0,\ x=0,\ d=-5\ln 4$ $x=5\ln\left(2+e^{2t}+e^{-2t}\right)-5\ln 4$ $x=5\ln\left(\dfrac{2+e^{2t}+e^{-2t}}{4}\right)$
(iii)	When $t=2, x=13.3$ m (3 s.f.). The squirrel has fallen 13.3 metres.
(iv)	Note that $y=\dfrac{dx}{dt}$ is the velocity of the falling squirrel at any time *t*. $y=\dfrac{10\left(e^{2t}-1\right)}{e^{2t}+1}$ $=\dfrac{10\left(1-e^{-2t}\right)}{1+e^{-2t}}$ As $t\to\infty,\ e^{-2t}\to 0,\ y\to 10$ Hence, the terminal velocity of the falling squirrel is 10 m/s.

Qn	Solution
11	**APGP**
(i)	(see below)

n (no. of years)	Total amount in account *A* at the end of n[th] year
1	$1000(1.035)$
2	$\left[1000+1000(1.035)\right]1.035$ $= 1000\left(1.035+1.035^2\right)$
3	$\left[1000\left(1.035+1.035^2\right)\right]1.035$ $=1000\left(1.035+1.035^2+1.035^3\right)$
	….
n	$1000\left(1.035+1.035^2+1.035^3+...+1.035^n\right)$

Total amount in account *A* at the end of *n* years

$= 1000\left(1.035+1.035^2+1.035^3+...+1.035^n\right)$

$= 1000\left(\dfrac{1.035\left(1-1.035^n\right)}{1-1.035}\right)$

$= 1000\left(\dfrac{1.035\left(1.035^n-1\right)}{1.035-1}\right)$

$= \dfrac{207000}{7}\left(1.035^n-1\right)$

Therefore, $p=\dfrac{207000}{7}$ and $q=1.035$.

(ii)

<u>Method 1:</u>

Using result from part (i),

When $n=23$, $\dfrac{207000}{7}\left(1.035^{23}-1\right)=35666.53<36000$

When $n=24$, $\dfrac{207000}{7}\left(1.035^{24}-1\right)=37949.86>36000$

When $n=23$, total amount in account *A* at the end of 2045 = $35666.53

Total amount in account *A* at the start of 2046

= $35666.53 + $1000 = $36666.53

Hence, the total amount in account *A* first exceed $36000 on **1 January 2046**.

<u>Method 2:</u>

n (no. of years)	Total amount in account *A* at the start of n[th] year
1	1000
2	$1000+1000(1.035)=1000(1+1.035)$
3	$\left[1000(1+1.035)\right]1.035+1000$ $=1000(1+1.035+1.035^2)$

	….
n	$1000\left(1+1.035+1.035^2+...+1.035^{n-1}\right)$

Total amount in account *A* at the start of the *n*th year

$= 1000\left(1+1.035+1.035^2+...+1.035^{n-1}\right)$

$= 1000\left(\dfrac{1\left(1-1.035^n\right)}{1-1.035}\right)$

$= \dfrac{200000}{7}\left(1.035^n-1\right)$

Using GC,

When $n=23$, $\dfrac{200000}{7}\left(1.035^{23}-1\right)=34460.41<36000$

When $n=24$, $\dfrac{200000}{7}\left(1.035^{24}-1\right)=36666.53>36000$

Since $n=24$, the total amount in account *A* first exceed \$36000 on **1 January 2046**.

(iii)

n (no. of years)	Total amount in account *B* at the end of nth year
1	$1000+40$
2	$(1000+40)+(1000+36)+40$ $=2(1000)+36+2(40)$
3	$\left[2(1000)+36+2(40)\right]+\left(1000+2(36)\right)+40$ $=3(1000)+\left(36+2(36)\right)+3(40)$
	….
n	$n(1000)+36\left[1+2+...+(n-1)\right]+n(40)$

Total amount in account *B* at the end of *n* years

$= n(1000)+36\left[1+2+...+(n-1)\right]+n(40)$

$= n(1040)+36\left[\dfrac{(n-1)n}{2}\right]$

$\dfrac{207000}{7}\left(1.035^n-1\right)>n(1040)+36\left[\dfrac{(n-1)n}{2}\right]$

$\dfrac{207000}{7}\left(1.035^n-1\right)-n(1040)+36\left[\dfrac{(n-1)n}{2}\right]>0$

Using GC,

When $n=6$,

$\dfrac{207000}{7}\left(1.035^6-1\right)-6(1040)+36\left[\dfrac{(6-1)6}{2}\right]=-0.592<0$

	When $n = 7$, $\frac{207000}{7}\left(1.035^7 - 1\right) - 7(1040) + 36\left[\frac{(7-1)7}{2}\right] = 15.687 > 0$ Least n = 7 Therefore, the total amount in account A first exceeds the total amount in account B in 2029.
(iv)	At end of 31 Dec 2032, $n = 10$ $10(1040) + k\left[\frac{(10-1)10}{2}\right] > \frac{207000}{7}\left(1.035^{10} - 1\right)$ $k > \left[\frac{207000\left(1.035^{10} - 1\right)}{7} - 10(1040)\right]\frac{1}{45}$ $k > 38.711$ Least $k = 39$ (to the nearest whole number)