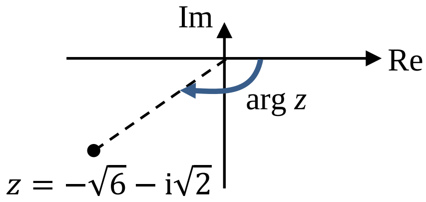
2022 H2 MATH (9758/02) JC 2 PRELIMINARY EXAMINATION SOLUTIONS

Qn	Solution
1	**Differentiation and applications**
(i)	$3y^2\frac{dy}{dx}-\left(y+x\frac{dy}{dx}\right)=2e^{2x}$ $\left(3y^2-x\right)\frac{dy}{dx}=2e^{2x}+y$ $\frac{dy}{dx}=\frac{2e^{2x}+y}{3y^2-x}$
(ii)	When $x=0$, $y^3-(0)y=e^{2(0)}+7$ $y^3=8$ $y=2$ $\frac{dy}{dx}=\frac{2e^{2(0)}+2}{3(2)^2-0}=\frac{4}{12}=\frac{1}{3}$ Equation of tangent to the curve at $x=0$: $y-2=\frac{1}{3}(x-0)$ $y=\frac{1}{3}x+2$

Qn	Solution
2	**Complex Numbers**

$$|z| = \sqrt{\left(-\sqrt{6}\right)^2 + \left(-\sqrt{2}\right)^2} = \sqrt{8} = 2\sqrt{2}$$

$$\begin{aligned}\arg z &= -\pi + \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{6}}\right) \\ &= -\pi + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \\ &= -\pi + \frac{\pi}{6} = -\frac{5\pi}{6}\end{aligned}$$



<u>Method 1</u>

$$\begin{aligned}\left|\frac{\mathrm{i}zw^2}{w^*}\right| &= \frac{(1)\left(2\sqrt{2}\right)(3)^2}{(3)} \\ &= 6\sqrt{2}\end{aligned}$$

$$\begin{aligned}\arg\left(\frac{\mathrm{i}zw^2}{w^*}\right) &= \arg(\mathrm{i}) + \arg z + 2\arg w - \arg\left(w^*\right) \\ &= \frac{\pi}{2} - \frac{5\pi}{6} + 2\left(-\frac{5\pi}{7}\right) - \left(\frac{5\pi}{7}\right) \\ &= -\frac{52\pi}{21} \\ &\equiv -\frac{10\pi}{21}\end{aligned}$$

<u>Method 2: Using exponential form</u>

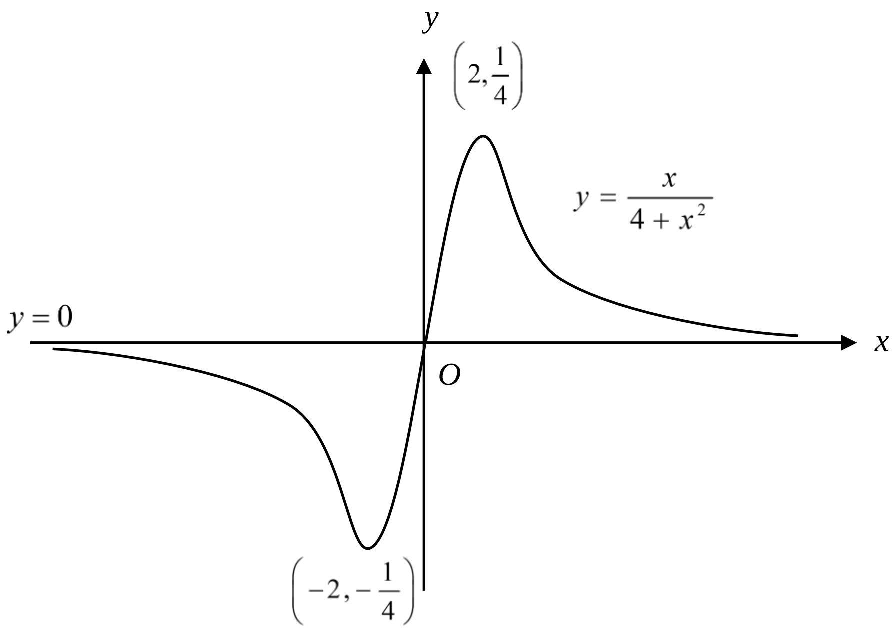
$$z = 2\sqrt{2}\mathrm{e}^{-\frac{5\pi}{6}\mathrm{i}}$$

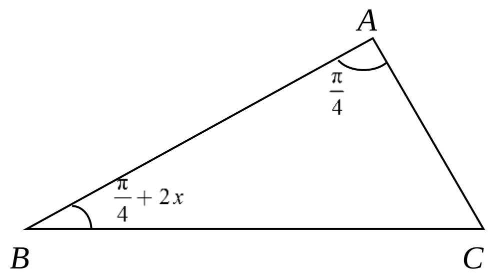
$$w = 3\left(\cos\frac{5\pi}{7} - \mathrm{i}\sin\frac{5\pi}{7}\right) = 3\mathrm{e}^{-\frac{5\pi}{7}\mathrm{i}}$$

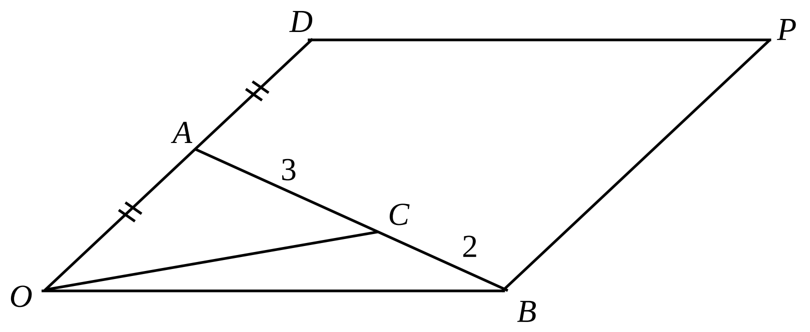
$$\begin{aligned}\frac{\mathrm{i}zw^2}{w^*} &= \frac{\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}\left(2\sqrt{2}\mathrm{e}^{\frac{-5\pi}{6}\mathrm{i}}\right)\left(3\mathrm{e}^{\frac{-5\pi}{7}\mathrm{i}}\right)^2}{3\mathrm{e}^{\frac{5\pi}{7}\mathrm{i}}} \\ &= 6\sqrt{2}\,\mathrm{e}^{\left(\frac{\pi}{2}-\frac{5\pi}{6}-\frac{10\pi}{7}-\left(\frac{5\pi}{7}\right)\right)\mathrm{i}} \\ &= 6\sqrt{2}\,\mathrm{e}^{\frac{-52\pi}{21}\mathrm{i}} \equiv 6\sqrt{2}\,\mathrm{e}^{\frac{-10\pi}{21}\mathrm{i}}\end{aligned}$$

$$\left|\frac{\mathrm{i}zw^2}{w^*}\right| = 6\sqrt{2}$$

$$\arg\left(\frac{\mathrm{i}zw^2}{w^*}\right) = -\frac{10\pi}{21}$$

Qn	Solution
3	**Graphing (Rational Function), Definite Integral (Volume)**
(i)	
(ii)	$\text{Vol} = \pi\left(\frac{1}{4}\right)^2(2) - \pi\int_0^2 \left(\frac{x}{4+x^2}\right)^2 \, dx$ $= \frac{\pi}{8} - \pi\int_0^2 \frac{x^2}{\left(4+x^2\right)^2} \, dx$ $= \frac{\pi}{8} - \pi\int_0^2 \frac{x^2}{\left(4+x^2\right)^2} \, dx$ $= \frac{\pi}{8} - \pi\int_0^{\frac{\pi}{4}} \frac{4\tan^2\theta}{\left(4+4\tan^2\theta^2\right)^2}\left(2\sec^2\theta\right) \, d\theta$ $= \frac{\pi}{8} - \pi\int_0^{\frac{\pi}{4}} \frac{4\tan^2\theta}{16\sec^4\theta}\left(2\sec^2\theta\right) \, d\theta$ $= \frac{\pi}{8} - \pi\int_0^{\frac{\pi}{4}} \frac{\tan^2\theta}{2\sec^2\theta} \, d\theta$ $= \frac{\pi}{8} - \frac{\pi}{2}\int_0^{\frac{\pi}{4}} \sin^2\theta \, d\theta$ $= \frac{\pi}{8} - \frac{\pi}{4}\int_0^{\frac{\pi}{4}} 1 - \cos 2\theta \, d\theta$ $= \frac{\pi}{8} - \frac{\pi}{4}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$ $= \frac{\pi}{8} - \frac{\pi}{4}\left[\left(\frac{\pi}{4} - \frac{1}{2}\right) - 0\right]$ $= \frac{\pi}{4} - \frac{\pi^2}{16}$

Qn	Solution
4	**Maclaurin Series**
	 $\angle ACB = \pi - \frac{\pi}{4} - \left(\frac{\pi}{4} + 2x\right) = \frac{\pi}{4} - 2x$ Using Sine Rule, $\frac{AB}{\sin\left(\frac{\pi}{2} - 2x\right)} = \frac{AC}{\sin\left(\frac{\pi}{4} + 2x\right)}$ $\frac{AB}{AC} = \frac{\sin\left(\frac{\pi}{2} - 2x\right)}{\sin\left(\frac{\pi}{4} + 2x\right)}$ $= \frac{\cos 2x}{\sin\frac{\pi}{4}\cos 2x + \cos\frac{\pi}{4}\sin 2x}$ $= \frac{\cos 2x}{\frac{1}{\sqrt{2}}\left(\cos 2x + \sin 2x\right)}$ $\frac{AB}{AC} = \frac{\sqrt{2}\cos 2x}{\cos 2x + \sin 2x}$ (shown)
	$\frac{AB}{AC} = \frac{\sqrt{2}\cos 2x}{\cos 2x + \sin 2x}$ $\approx \frac{\sqrt{2}\left(1 - \frac{(2x)^2}{2!}\right)}{\left(1 - \frac{(2x)^2}{2!}\right) + (2x)}$ $= \frac{\sqrt{2}\left(1 - 2x^2\right)}{1 + 2x - 2x^2}$ $= \sqrt{2}\left(1 - 2x^2\right)\left(1 + 2x - 2x^2\right)^{-1}$ $= \sqrt{2}\left(1 - 2x^2\right)\left(1 + (-1)\left(2x - 2x^2\right) + \frac{(-1)(-2)}{2!}\left(2x - 2x^2\right)^2 + \ldots\right)$ $= \sqrt{2}\left(1 - 2x^2\right)\left(1 - 2x + 2x^2 + 4x^2 + \ldots\right)$ $= \sqrt{2}\left(1 - 2x^2\right)\left(1 - 2x + 6x^2 + \ldots\right)$ $= \sqrt{2}\left(1 - 2x + 6x^2 - 2x^2 + \ldots\right)$ $= \sqrt{2}\left(1 - 2x + 4x^2 + \ldots\right)$

Qn	Solution
5	**Vectors**
	
(i)	$\overrightarrow{OC}=\frac{2\overrightarrow{OA}+3\overrightarrow{OB}}{5}=\frac{2}{5}\mathbf{a}+\frac{3}{5}\mathbf{b}$ $\overrightarrow{OP}=\overrightarrow{OD}+\overrightarrow{OB}=2\overrightarrow{OA}+\overrightarrow{OB}=2\mathbf{a}+\mathbf{b}$ Area of triange $OPC=\frac{1}{2}\left\|\overrightarrow{OP}\times\overrightarrow{OC}\right\|$ $=\frac{1}{2}\left\|(2\mathbf{a}+\mathbf{b})\times\left(\frac{2}{5}\mathbf{a}+\frac{3}{5}\mathbf{b}\right)\right\|$ $=\frac{1}{10}\left\|2\mathbf{a}\times2\mathbf{a}+2\mathbf{a}\times3\mathbf{b}+\mathbf{b}\times2\mathbf{a}+\mathbf{b}\times3\mathbf{b}\right\|$ $=\frac{1}{10}\left\|6\mathbf{a}\times\mathbf{b}-2\mathbf{a}\times\mathbf{b}\right\|$ $=\frac{2}{5}\left\|\mathbf{a}\times\mathbf{b}\right\|$ $\therefore k=\frac{2}{5}$
(ii)	Line *AB*: $\mathbf{r}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a}),\ \lambda\in\mathbb{R}$ Line *OP*: $\mathbf{r}=\mu(2\mathbf{a}+\mathbf{b}),\ \mu\in\mathbb{R}$ To find intersection point: $\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})=\mu(2\mathbf{a}+\mathbf{b})$ $(1-\lambda)\mathbf{a}+\lambda\mathbf{b}=2\mu\mathbf{a}+\mu\mathbf{b}$ Since , $\mathbf{a}$ is not parallel to $\mathbf{b}$ and they are nonzero vectors, $1-\lambda=2\mu$ $\lambda=\mu$ Solving, $\lambda=\mu=\frac{1}{3}$ $\overrightarrow{OE}=\frac{1}{3}(2\mathbf{a}+\mathbf{b})$

(iii)	**Method 1:** Since E is the foot of perpendicular from D to the line OP, $\overrightarrow{DE}\cdot\overrightarrow{OP}=0$ $\left(\frac{1}{3}(2\mathbf{a}+\mathbf{b})-2\mathbf{a}\right)\cdot(2\mathbf{a}+\mathbf{b})=0$ $\frac{1}{3}(\mathbf{b}-4\mathbf{a})\cdot(2\mathbf{a}+\mathbf{b})=0$ $\mathbf{b}\cdot 2\mathbf{a}+\mathbf{b}\cdot\mathbf{b}-4\mathbf{a}\cdot 2\mathbf{a}-4\mathbf{a}\cdot\mathbf{b}=0$ $\lvert\mathbf{b}\rvert^2-8\lvert\mathbf{a}\rvert^2-2\mathbf{a}\cdot\mathbf{b}=0$ $2\mathbf{a}\cdot\mathbf{b}=\lvert\mathbf{b}\rvert^2-8\lvert\mathbf{a}\rvert^2$ $2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos A\hat{O}B=\lvert\mathbf{b}\rvert^2-8\lvert\mathbf{a}\rvert^2$ $\cos A\hat{O}B=\frac{\lvert\mathbf{b}\rvert^2-8\lvert\mathbf{a}\rvert^2}{2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ $=\frac{\lvert\mathbf{b}\rvert^2-8}{2\lvert\mathbf{b}\rvert}$ $\quad(\because \mathbf{a}$ is a unit vector$)$ Since $A\hat{O}B$ is acute, $\frac{\lvert\mathbf{b}\rvert^2-8}{2\lvert\mathbf{b}\rvert}>0$ and since $\lvert\mathbf{b}\rvert>0$, $\lvert\mathbf{b}\rvert>2\sqrt{2}$ Also, $\frac{\lvert\mathbf{b}\rvert^2-8}{2\lvert\mathbf{b}\rvert}<1$ $\quad(\because \mathbf{a}$ is not parallel to $\mathbf{b})$ $\lvert\mathbf{b}\rvert^2-8<2\lvert\mathbf{b}\rvert$ $\quad(\because \lvert\mathbf{b}\rvert>0)$ $\lvert\mathbf{b}\rvert^2-2\lvert\mathbf{b}\rvert-8<0$ $(\lvert\mathbf{b}\rvert-4)(\lvert\mathbf{b}\rvert+2)<0$ we get $0<\lvert\mathbf{b}\rvert<4$. Thus, $2\sqrt{2}<\lvert\mathbf{b}\rvert<4$

	Method 2: Since E is the foot of perpendicular from D to the line OP, $\overrightarrow{DE}\cdot\overrightarrow{OP}=0$ $\left(\frac{1}{3}(2\mathbf{a}+\mathbf{b})-2\mathbf{a}\right)\cdot(2\mathbf{a}+\mathbf{b})=0$ $\frac{1}{3}(\mathbf{b}-4\mathbf{a})\cdot(2\mathbf{a}+\mathbf{b})=0$ $\mathbf{b}\cdot 2\mathbf{a}+\mathbf{b}\cdot\mathbf{b}-4\mathbf{a}\cdot 2\mathbf{a}-4\mathbf{a}\cdot\mathbf{b}=0$ $\lvert\mathbf{b}\rvert^2-8\lvert\mathbf{a}\rvert^2-2\mathbf{a}\cdot\mathbf{b}=0$ $2\mathbf{a}\cdot\mathbf{b}=\lvert\mathbf{b}\rvert^2-8\lvert\mathbf{a}\rvert^2$ $2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos A\hat{O}B=\lvert\mathbf{b}\rvert^2-8\lvert\mathbf{a}\rvert^2$ $\lvert\mathbf{b}\rvert^2-2\lvert\mathbf{b}\rvert\cos A\hat{O}B-8=0 \qquad (\because \mathbf{a} \text{ is a unit vector})$ $\lvert\mathbf{b}\rvert=\dfrac{2\cos A\hat{O}B\pm\sqrt{4\cos^2 A\hat{O}B-4(1)(-8)}}{2}$ $=\cos A\hat{O}B\pm\sqrt{\cos^2 A\hat{O}B+8}$ $=\cos A\hat{O}B+\sqrt{\cos^2 A\hat{O}B+8} \quad \text{or} \quad \cos A\hat{O}B-\sqrt{\cos^2 A\hat{O}B+8}$ Since $\lvert\mathbf{b}\rvert>0$, $\lvert\mathbf{b}\rvert=\cos A\hat{O}B+\sqrt{\cos^2 A\hat{O}B+8}$ Since $A\hat{O}B$ is acute, $0<\cos A\hat{O}B<1$ $0<\cos^2 A\hat{O}B<1$ And $\cos A\hat{O}B$ is strictly decreasing for the given domain, We have $2\sqrt{2}<\lvert\mathbf{b}\rvert<4$

Qn	Solution
6	**Discrete Random Variable**
(i)	Probability Distribution of X

x	0	1	2	3	4	5	6
$\mathrm{P}(X=x)$	p	q	$\frac{q}{3}$	q	$\frac{q}{3}$	q	$\frac{q}{3}$

$$\mathrm{E}(X)=2$$

$$0+q+\frac{2q}{3}+3q+\frac{4q}{3}+5q+\frac{6q}{3}=2$$

$$p+q+\frac{q}{3}+q+\frac{q}{3}+q+\frac{q}{3}=1$$

Solving, $p=\frac{5}{13}$, $q=\frac{2}{13}$

Probability Distribution of X

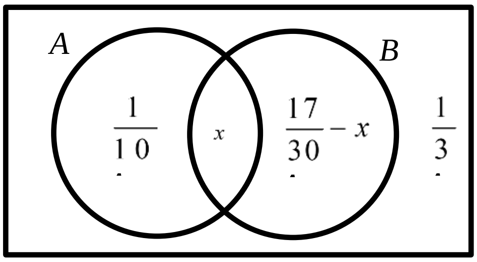
x	0	1	2	3	4	5	6
$\mathrm{P}(X=x)$	$\frac{5}{13}$	$\frac{2}{13}$	$\frac{2}{39}$	$\frac{2}{13}$	$\frac{2}{39}$	$\frac{2}{13}$	$\frac{2}{39}$

(ii)

$$\mathrm{P}(X_1+X_2=4)=\mathrm{P}(X_1=0,X_2=4)+\mathrm{P}(X_1=4,X_2=0)+\mathrm{P}(X_1=1,X_2=3)+\mathrm{P}(X_1=3,X_2=1)+\mathrm{P}(X_1=2,X_2=2)$$

$$=\frac{5}{13}\left(\frac{2}{39}\right)\times 2+\frac{2}{13}\left(\frac{2}{13}\right)\times 2+\frac{2}{39}\left(\frac{2}{39}\right)$$

$$=\frac{136}{1521}$$ or 0.0894 (3 s.f.)

Qn	Solution
7	**Permutations & Combinations and Probability**
(a)(i)	Number of teams $= {}^{10}C_5 = 252$
(a)(ii)	Number of teams $= {}^{6}C_4 + \left({}^{3}C_1\right)\left({}^{6}C_3\right) = 75$
(b)(i)	**Method 1:**  Let $\mathrm{P}(A \cap B) = x$ $\mathrm{P}(B \mid A) = \frac{4}{5}$ $\frac{x}{\frac{1}{10} + x} = \frac{4}{5}$ $x = \frac{2}{5}$
	Method 2: $\mathrm{P}(A \cup B) = 1 - \mathrm{P}(A' \cap B') = \frac{2}{3}$ $\mathrm{P}(B \mid A) = \frac{4}{5} = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)} \Rightarrow \mathrm{P}(A \cap B) = \frac{4}{5}\mathrm{P}(A)$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $= \mathrm{P}(A) + \frac{17}{30} - \frac{4}{5}\mathrm{P}(A)$ $= \frac{1}{5}\mathrm{P}(A) + \frac{17}{30}$ $\Rightarrow \mathrm{P}(A) = \frac{1}{2}$ $\Rightarrow \mathrm{P}(A \cap B) = \frac{2}{5}$

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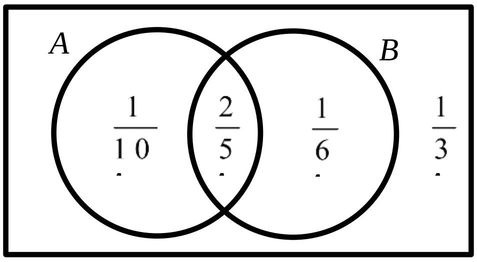
<td>(b)(ii)</td>
<td>


$$\mathrm{P}(A \mid B') = \frac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')}$$

$$= \frac{\mathrm{P}(A \cap B')}{1 - \mathrm{P}(B)}$$

$$= \frac{\frac{1}{10}}{1 - \frac{17}{30}}$$

$$= \frac{3}{13}$$
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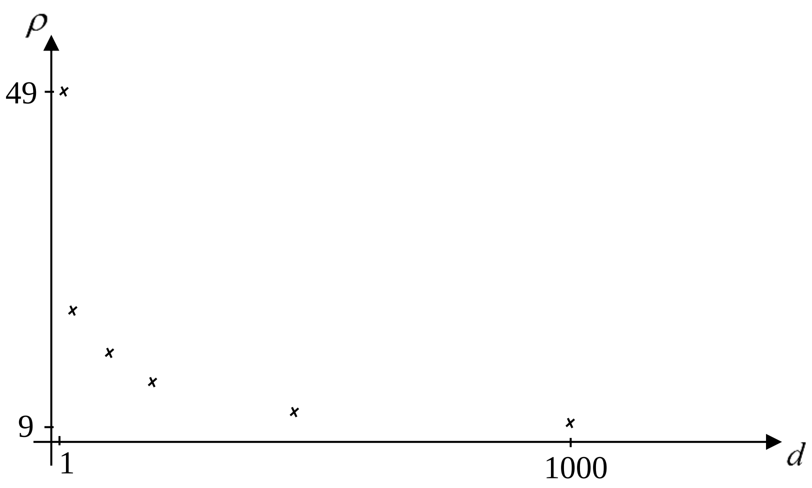
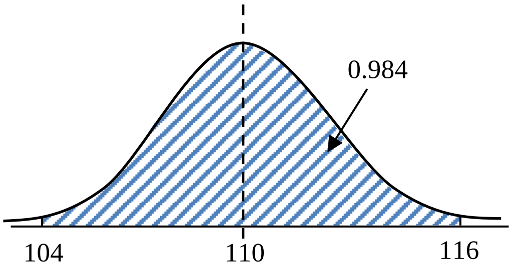
Qn	Solution
8	**Correlation and Regression**
(i)	
(ii)(a)	$r = -0.62016 \approx -0.620$
(ii)(b)	$r = -0.99371 = -0.994$
(iii)	Based on the scatter diagram, as d increases, ρ decreases at a decreasing rate. Also, $\lvert r \rvert = \lvert -0.99371 \rvert = 0.994$ for $\ln\rho$ and $\ln d$ is closer to 1 as compared to $\lvert r \rvert = \lvert -0.62016 \rvert = 0.620$ for ρ and d. Hence, the relationship between ρ and d is better modelled by $\ln\rho = A + B\ln d$. $\ln\rho = 3.8793 - 0.25338\ln d \approx 3.88 - 0.253\ln d$, where $A = 3.88, B = -0.253$
(iv)	$\ln\rho = 3.8793 - 0.25338\ln d$ When $\rho = 8$, $\ln 8 = 3.8793 - 0.25338\ln d$ $d = 1216.1 \approx 1216$ mm (to nearest integer) Even though $\lvert r \rvert = 0.994$ is close to 1, since $\rho = 8$ lies outside the data range of ρ, the linear relation may no longer hold, hence the estimate is not reliable.
(v)	The product moment correlation coefficient will be the **same**, as r is **independent of the scale of measurement**.

Qn	Solution
9	**Hypothesis Testing**
(i)	Since the weights of apples should be **close to the mean of 200g**, using $\sum(x-200)^2$ instead of $\sum x^2$ ensures the sum is manageable and **not too large**. **OR** By coding the summarised data of $(x-200)$ with reference to the mean of 200, it reduces the value of summarized data into a number that can be more easily handled when finding unbiased estimates.
(ii)	An unbiased estimate for the population mean is $\bar{x}=\frac{-30}{30}+200=199$ An unbiased estimate for the population variance is $s^2=\frac{1}{29}\left(1800-\frac{(-30)^2}{30}\right)=\frac{1770}{29}$
(iii)	Let μ be the population mean weight of apples, in g. $\text{H}_0:\mu=200$ $\text{H}_1:\mu<200$ Under H_0, Since $n=30$ is large, by Central Limit Theorem, $\bar{X}\sim \text{N}\left(200,\frac{1770}{(29)(30)}\right)$ approximately. Test Statistic: $Z=\dfrac{\bar{X}-200}{\sqrt{\frac{1770}{(29)(30)}}}$ Level of significance: 10% Reject H_0 if p-value < 0.1. Under H_0, using GC, $p\text{-value}=0.24162\ (5\text{ s.f})=0.242\ (3\text{ s.f})$ Since $p\text{-value}=0.242>0.1$, we do not reject H_0 and conclude that there is insufficient evidence, at 10% level of significance, that the population mean weight of apples sold by the fruit seller is less than 200g. Thus the fruit's claim is valid.

(iv)	Let Y be the weight of a randomly chosen orange, in g. Let μ_y be the population mean weight of oranges, in g. $H_0 : \mu_y = 120$ $H_1 : \mu_y \neq 120$ Under H_0, $Y \sim N(120, 8^2)$ $\Rightarrow$ $\overline{Y} \sim N\left(120, \frac{8^2}{30}\right)$ Test Statistic: $Z = \dfrac{\overline{Y} - 120}{8/\sqrt{30}}$ Level of significance: 10% Reject H_0 if z-value < -1.6449 or z-value > 1.6449 Since H_0 is rejected, $\dfrac{\overline{y} - 120}{8/\sqrt{30}} < -1.6449$ or $\dfrac{\overline{y} - 120}{8/\sqrt{30}} > 1.6449$ $\overline{y} < 117.60$ or $\overline{y} > 122.40$ $\overline{y} < 117$ or $\overline{y} > 123$ $\{\overline{y} \in \mathbb{R}^+ : \overline{y} < 117 \text{ or } \overline{y} > 123\}$ or $\{\overline{y} \in \mathbb{R} : 0 < \overline{y} < 117 \text{ or } \overline{y} > 123\}$

Qn	Solution
10	**Binomial Distribution**
(i)	The probability that a randomly chosen key chain is defective remains constant at 0.03 for all key chains in a box. Whether a randomly chosen key chain is defective is independent of any other key chains in a box.
(ii)	Let X be the number of defective key chains out of n key chains in a box. $X \sim \mathrm{B}(n, 0.03)$ $\mathrm{P}(X \le 2) < 0.95$ when $n = 27$, $\mathrm{P}(X \le 2) = 0.9538 > 0.95$ when $n = 28$, $\mathrm{P}(X \le 2) = 0.9494 < 0.95$ Least value of n = 28
(iii)	**Method A:** Let Y be the number of defective key chains out of 20 key chains in a box. $Y \sim \mathrm{B}(20, 0.03)$ $\mathrm{P}(Y \le 2) = 0.97899 = 0.979$ (3 s.f.) **Method B:** Let W be the number of defective key chains out of 10 key chains in a box. $W \sim \mathrm{B}(10, 0.03)$ P(a batch is accepted) = $\mathrm{P}(W = 0) + P(W = 1)P(W \le 1)$ = 0.95762 = 0.958 (3 s.f.)
(iv)	Expected number for Method A $= 20$ Expected number for Method B $= 10 \times \left(1 - \mathrm{P}(W = 1)\right) + 20 \times \mathrm{P}(W = 1)$ $= 12.3$ (3 s.f.) Since the expected number of keychains to be sampled for method B is lower, the company might choose B instead of A as it saves time in checking (or any other valid reason).
(v)	$P(Y \ge 3) = 1 - P(Y \le 2) = 1 - 0.97899 = 0.02101$ Let S be the number of boxes with 3 or more defective key chains out of 30 boxes. $S \sim B(30, 0.02101)$ Let T be the number of boxes with 3 or more defective key chains out of 14 boxes. $T \sim B(14, 0.02101)$ Required probability = $\dfrac{P(T = 2) \times 0.02101 \times (0.97899)^{15}}{P(S = 3)}$ = 0.0224

Qn	Solution
11	**Normal and Sampling Distribution**

(i)	Let X and Y be the volume of oil in a randomly chosen barrel of light and heavy oil respectively. $X \sim \mathrm{N}\left(110,\ 2.5^2\right)$ $\quad$ $Y \sim \mathrm{N}\left(145,\ 3.5^2\right)$ $\mathrm{P}(104 < X < 116) = 0.984$
(ii)	
(iii)	Required Probability $= \mathrm{P}(142 < Y < 150)^4 \times \mathrm{P}(Y > 150) \times \mathrm{P}(Y < 142)^2 \times \dfrac{7!}{4!2!}$ $= 0.0863$
(iv)	Let $\overline{Y} = \dfrac{Y_1 + Y_2 + \ldots + Y_n}{n}$ and $\overline{Y} \sim \mathrm{N}\left(145, \dfrac{3.5^2}{n}\right)$ $\mathrm{P}\left(\overline{Y} > k\right) \geq 0.3$ $\mathrm{P}\left(Z > \dfrac{k-145}{\frac{3.5}{\sqrt{n}}}\right) \geq 0.3$ $\dfrac{k-145}{\frac{3.5}{\sqrt{n}}} \leq 0.52440$ $k \leq 145 + \dfrac{1.84}{\sqrt{n}}$
(v)	Let $T = 0.83\left(X_1 + X_2 + \cdots X_{25}\right) + 0.94\left(Y_1 + Y_2 + \cdots Y_{30}\right)$ $\mathrm{E}(T) = 0.83(110 \times 25) + 0.94(145 \times 30) = 6371.5$ (exact) $\mathrm{Var}(T) = 0.83^2\left(2.5^2 \times 25\right) + 0.94^2\left(3.5^2 \times 30\right) = 432.363625$ (exact) $T \sim \mathrm{N}(6371.5, 432.363625)$ (exact) $\mathrm{P}\left(T + 25(5) + 30(8) > 6800\right)$ $= \mathrm{P}(T > 6435)$ $= 0.00113$
(vi)	The distributions of the volume of **all types of oil** are **independent** of one another.