TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME**: _____**

CIVICS GROUP: **_____**

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H2 MATHEMATICS   **9758/02**
Paper 2   19 SEPTEMBER 2022
                                                         3 hours

Candidates answer on the question paper.

Additional material: List of Formulae (MF26)

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READ THESE INSTRUCTIONS FIRST

Write your name and Civics Group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or
1 decimal place in the case of angles in degrees, unless a different
level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless
a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not
allowed in a question, you are required to present the mathematical
steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your
answers.

For Examiners' Use	
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Total	

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

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This document consists of 6 printed pages and 0 blank pages.

[Turn Over

Section A: Pure Mathematics [40 marks]

1 The equation of a curve is given by $y^3 - xy = e^{2x} + 7$.

    (i) Find $\dfrac{dy}{dx}$ in terms of x and y.   [2]

    (ii) Find the equation of the tangent to the curve at the point where $x = 0$.   [2]

2 It is given that $z = -\sqrt{6} - i\sqrt{2}$ and $w = 3\left(\cos\dfrac{5\pi}{7} - i\sin\dfrac{5\pi}{7} \right)$. Without the use of a

calculator, find the modulus and argument of $\dfrac{izw^2}{w*}$ in exact form.   [6]

3 A curve C has equation $y = \dfrac{x}{4 + x^2}$.

    (i) Sketch the graph of C, stating the equations of any asymptotes and the coordinates of any turning points.   [2]

    (ii) The region bounded by the curve, the y-axis and the line $y = \dfrac{1}{4}$ is rotated about the x-axis through $360°$. Use the substitution $x = 2\tan\theta$ to find the exact volume of the solid obtained.   [5]

4 In the triangle ABC, angle $BAC = \dfrac{\pi}{4}$ radians and angle $ABC = \left(\dfrac{\pi}{4} + 2x \right)$ radians. Show

that $\dfrac{AB}{AC} = \dfrac{\sqrt{2}\cos 2x}{\cos 2x + \sin 2x}$.   [3]

Given that x is sufficiently small for which x^3 and higher powers of x can be ignored, find the series expansion of $\dfrac{AB}{AC}$ in ascending powers of x.   [4]

5 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively where $\mathbf{a}$ and $\mathbf{b}$ are non-zero non-parallel vectors. The point C is on AB such that $AC:CB = 3:2$ and the point D is such that A is the mid-point of OD. It is also given that $ODPB$ forms a parallelogram.

(i) By finding $\overrightarrow{OC}$ and $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, show that the area of triangle OPC can be written as $k\left|\mathbf{a}\times\mathbf{b}\right|$, where k is a constant to be determined. [5]

The lines AB and OP intersect at point E.

(ii) Find the position vector of E in terms of $\mathbf{a}$ and $\mathbf{b}$. [4]

(iii) It is given further that angle AOB is acute and $\mathbf{a}$ is a unit vector. Find the range of values of $\left|\mathbf{b}\right|$ such that E is the foot of perpendicular from D to the line OP, giving your answers in exact form. [7]

Section B: Probability and Statistics [60 marks]

6 The probability function of a discrete random variable, X, is given as follows:

$$P(X = x) = \begin{cases} p & \text{if } x = 0 \\ \dfrac{1}{3}P(X = x-1) & \text{if } x = 2,4,6 \\ q & \text{if } x = 1,3,5 \end{cases}$$

(i) Given that the expected value of X is 2, find the probability distribution of X. [4]

(ii) Given that X_1 and X_2 are two independent observations of X, find $P(X_1 + X_2 = 4)$. [2]

7 (a) A team of 5 people from a family of 7 adults and 3 children is to be selected for a competition. Find the number of teams that can be selected if

(i) there are no restrictions, [1]

(ii) at most one child and the oldest adult must be in the team. [2]

(b) For events A and B, it is given that $P(B) = \dfrac{17}{30}$, $P(A' \cap B') = \dfrac{1}{3}$ and $P(B \mid A) = \dfrac{4}{5}$.

(i) Find $P(A \cap B)$. [3]

(ii) Find $P(A \mid B')$. [2]

[Turn Over

8 To improve the prediction accuracy of earthquakes, a group of seismologists gathered the following information about the crack density ρ, measured in millimetres^{-1}, at different distances away from the fault line d, measured in millimetres.

d	1	10	100	200	500	1000
ρ	49	27	16	11	10	9

  (i) Sketch a scatter diagram of the data. [1]

 (ii) Find the product moment correlation coefficient between

    (a) ρ and d,

    (b) $\ln \rho$ and $\ln d$. [2]

(iii) Using the answers to parts **(i)** and **(ii)**, explain why the relationship between ρ and d is better modelled by $\ln \rho = A + B \ln d$, as compared to $\rho = C + Dd$, where *A*, *B*, *C* and *D* are real constants. Hence, find the equation of a suitable regression line for this better model. [3]

(iv) Use the regression line found in part **(iii)** to estimate the distance away from the fault line, to the nearest integer, when the crack density is 8 mm^{-1} and comment on its reliability. [2]

 (v) Without further calculations, explain whether the product moment correlation coefficient between $\ln \rho$ and $\ln d$ would be different if d was recorded in metres instead. [1]

9 A fruit seller claims that the apples he sells have a mean weight of 200g. A consumer believes the fruit seller is overstating his claim and decides to do a hypothesis test. He buys a random sample of 30 apples from the fruit seller and measures x, the weight of each apple with the following results:

$$\sum(x-200)=-30 \text{ and } \sum(x-200)^2 =1800.$$

(i) Explain why, in this context, the given data is summarised in terms of $(x-200)$ rather than x. [1]

(ii) Find unbiased estimates for the population mean and variance. [2]

(iii) Test, at the 10% significance level, whether the fruit seller has overstated the mean weight of apples that he sells. [4]

The fruit seller wishes to test whether the weight of oranges he sells has a mean weight of 120g and it is given that the weights of oranges sold by the fruit seller are normally distributed with a standard deviation of 8g.

(iv) For a random sample of 30 oranges, find the set of possible values of the sample mean weight to conclude that the mean weight of oranges is not 120g at the 10% level of significance. (Answer obtained by trial and improvement from a calculator will obtain no marks.) [4]

10 A company produces large batches of key chains. It is known that, on average, 3% of the key chains are defective. The key chains are packed in boxes of n.

  (i) State, in context, two assumptions needed for the number of defective key chains in a box to be well-modelled by a binomial distribution. [2]

  Assume that the assumptions stated above hold.

  (ii) The probability that a box contains fewer than 3 defective key chains is less than 0.95. Find the smallest possible integer value of n. [2]

As part of the company's quality control process, the company is considering 2 methods for inspection of a batch of key chains.

Method A

  The key chains are packed in boxes of 20. A box is randomly selected. If there are 2 or fewer defective key chains, the batch will be accepted, otherwise the batch will be rejected.

Method B

  The key chains are packed in boxes of 10. A box is randomly selected. The batch is accepted if there are no defective key chains and rejected if there are 2 or more defective key chains. Otherwise, randomly select another box for inspection. If there are fewer than 2 defective key chains in the second box, the batch will be accepted.

  (iii) For each model, find the probability that the batch will be accepted. [3]

  (iv) By calculating the expected number of keychains to be sampled for each method, give a possible reason why the company would choose method B. [2]

The key chains are packed in boxes of 20.

  (v) A random sample of 30 boxes is taken. Given that there are 3 boxes with 3 or more defective key chains, find the probability that the third box with 3 or more defective key chains occurs on the 15th box. [3]

11 **In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

In a factory, oil is stored in barrels of different sizes. It is given that the volumes of oil in the barrels can be modelled using normal distributions with means and standard deviations as shown in the table.

	Mean volume (in litres)	Standard deviation (in litres)
Light oil	110	2.5
Heavy oil	145	3.5

(i) Find the probability that the volume of light oil in a randomly chosen barrel is between 104 litres and 116 litres. [1]

(ii) Sketch the distribution for the volume of light oil, indicating clearly the probability found in part **(i)**. [2]

(iii) A random sample of seven barrels of heavy oil is chosen. Find the probability that exactly four barrels of heavy oil have volume between 142 and 150 litres each and exactly one barrel of heavy oil has volume more than 150 litres. [3]

(iv) A random sample of n barrels of heavy oil is chosen and it is given that the probability that the mean volume of these n barrels of heavy oil exceeding k litres is at least 0.3. Find an inequality, expressing k in terms of n. [3]

A lorry with a maximum laden mass of 6800 kilograms is used to transport 25 barrels of light oil and 30 barrels of heavy oil. It is given that the densities of light oil and heavy oil are 0.83 kilograms per litre and 0.94 kilograms per litre respectively, and the empty barrels for light oil weigh 5 kilograms each and the empty barrels for heavy oil weigh 8 kilograms each.

(v) Find the probability that the load of the lorry exceeds its maximum laden mass. [4]

[Density is defined as $\dfrac{\text{Mass}}{\text{Volume}}$.]

(vi) State an assumption needed for your calculations in part **(v)**. [1]

End of Paper