EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2019

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME

CLASS

INDEX NO.

MATHEMATICS

9758/02

18 September 2019

Paper 2 [100 marks]

3 hours

Candidates answer on the Question Paper

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and question number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.
Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **25** printed pages (including this cover page) and **1** blank page.

For markers' use:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total

Section A: Pure Mathematics [40 marks]

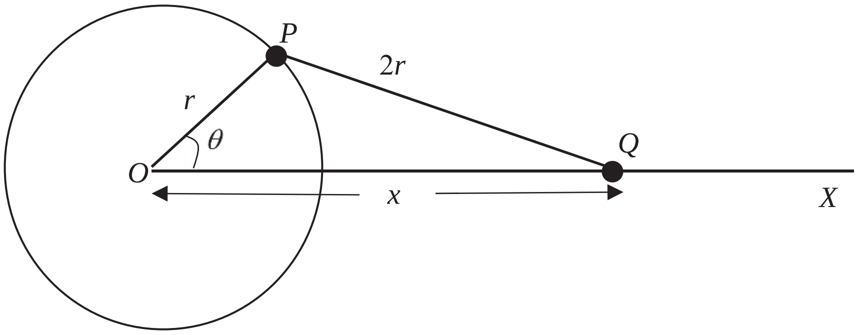
1 **(a)** Find the complex numbers z and w that satisfy the equations

$$\frac{z}{w} = 2 + 2\text{i},$$

$$(1 - 2\text{i})z = 39 - (11\text{i})w. \quad [3]$$

(b) It is given that $(1 + \text{i}c)^3$ is real, where c is also real. By first expressing $(1 + \text{i}c)^3$ in Cartesian form, find all possible values of c. [3]

2



The diagram shows a mechanism for converting rotational motion into linear motion. The point P is on the circumference of a disc of fixed radius r which can rotate about a fixed point O. The point Q can only move on the line OX, and P and Q are connected by a rod of length $2r$. As the disc rotates, the point Q is made to slide along OX. At time t, angle POQ is θ and the distance OQ is x.

(i) State the maximum and minimum values of x. [1]

(ii) Show that $x = r\left(\cos\theta + \sqrt{4 - \sin^2\theta}\right)$. [2]

(iii) At a particular instant, $\theta = \frac{\pi}{6}$ and $\frac{\text{d}\theta}{\text{d}t} = 0.3$. Find the numerical rate at which point Q is moving towards point O at that instant, leaving your answer in terms of r. [3]

3 The position vectors of points P and Q, with respect to the origin O, are $\mathbf{p}$ and $\mathbf{q}$ respectively. Point R, with position vector $\mathbf{r}$, is on PQ produced, such that $3\overrightarrow{PR} = 5\overrightarrow{PQ}$.

(i) Given that $|\mathbf{p}| = \sqrt{29}$ and $\mathbf{p.r} = 11$, find the length of projection of $\overrightarrow{OQ}$ onto $\overrightarrow{OP}$. [4]

(ii) S is another point such that $\overrightarrow{PS} = \mathbf{r}$. Given that $\mathbf{p} = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ and $\mathbf{r} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$, find the area of the quadrilateral $OPSR$. [3]

4 The ceiling function maps a real number *x* to the least integer greater than or equal to *x*. Denote the ceiling function as $\lceil x \rceil$. For example, $\lceil 2.1 \rceil = 3$ and $\lceil -3.8 \rceil = -3$.

The function f is defined by

$$\mathrm{f}(x) = \begin{cases} \lceil x \rceil & \text{for } x \in \mathbb{R}, \quad -2 < x \le 1, \\ 0 & \text{for } x \in \mathbb{R}, \quad 1 < x \le 2. \end{cases}$$

(i) Find the value of $\mathrm{f}(-1.4)$. [1]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ for $-2 < x \le 2$. [2]

(iii) Does f^{-1} exist? Justify your answer. [1]

(iv) Find the range of f. [1]

The function g is defined as $\mathrm{g} : x \mapsto \dfrac{ax-3}{x-a}$, $x \in \mathbb{R}, x \ne a$, where $a > 0, a \ne 3$.

(v) Find $\mathrm{g}^2(x)$. Hence, or otherwise, evaluate $\mathrm{g}^{2019}(5)$, leaving your answer in *a* if necessary. [4]

(vi) Given that $a = 3$, find the range of gf. [1]

5 **(a)** The r^{th} term of a sequence is given by $u_r = \dfrac{4}{M^{3r-1}}$, where $M > 1$.

(i) Write down the first three terms of u_r in terms of *M*. [1]

(ii) Show that $\displaystyle\sum_{r=1}^{n} u_r = \frac{4M}{M^3 - 1}\left(1 - \frac{1}{M^{3n}}\right)$. [2]

(iii) Give a reason why the series in **(ii)** is convergent and state the sum to infinity. [2]

(b) **(i)** Show that $\cos\left(\dfrac{2r+1}{2}\right) - \cos\left(\dfrac{2r-1}{2}\right) = -2\sin\left(\dfrac{1}{2}\right)\sin(r)$. [2]

(ii) Hence show that $\displaystyle\sum_{r=1}^{n} \sin r = \operatorname{cosec}\left(\frac{1}{2}\right)\sin\frac{n+1}{2}\sin\frac{n}{2}$. [4]

Section B: Probability and Statistics [60 marks]

6 A biased tetrahedral die has four faces, marked with the numbers 1, 2, 3 and 4. On any throw, the probability of the die landing on each face is shown in the table below, where c and d are real numbers.

Number on face	1	2	3	4
Probability of landing on face	0.3	c	d	0.2

(i) Write down an expression for d in terms of c. [1]

(ii) By writing the variance of the result of one throw of the die in the form $-\alpha(c-h)^2+k$, where α, h and k are positive constants to be determined, find the value of c which maximises this variance. [5]

(iii) If $c=0.2$, find the probability that in 10 throws of the die, at least 7 throws land on an even number. [2]

7 $X_1, X_2, X_3, \ldots$ are independent normally-distributed random variables with common mean μ and **different** variances. For each positive integer n, $\text{Var}(X_n)=2n$.

(i) Find $\text{P}(\mu-1<X_2<\mu+1)$. [3]

(ii) Find $\text{P}(X_3 \geq X_4)$. [1]

(iii) For each n, let $Y_n=\dfrac{X_1+X_2+\ldots+X_n}{n}$. By finding the distribution of Y_n in terms of n, determine the smallest integer value of n such that $\text{P}(\mu-1<Y_n<\mu+1)>\dfrac{2}{3}$. [4]

8 For events *A* and *B*, it is given that $\text{P}(A)=\dfrac{2}{5}$, $\text{P}(A\cup B)=\dfrac{6}{7}$, and $\text{P}(A\cap B')=\dfrac{1}{3}$. Find:

(i) $\text{P}(B)$; [2]

(ii) $\text{P}(A'\mid B)$. [2]

A third event, *C*, is such that *B* and *C* are independent, and $\text{P}(C)=\dfrac{2}{5}$.

(iii) Find $\text{P}(B'\cap C)$. [2]

(iv) Hence, find the greatest and least possible values of $\text{P}(A\cap B'\cap C)$. [4]

9 **(a)** Find the number of ways of arranging the letters of the word JEWELLERY, if:

(i) there are no restrictions. [1]

(ii) the arrangement starts with 'L', and between any two 'E's there must be at least 2 other letters. [3]

A 4-letter 'codeword' is formed by taking an arrangement of 4 letters from the word JEWELLERY.

(iii) Find the number of 4-letter codewords that can be formed. [3]

(b) Mr and Mrs Lee, their three children, and 5 others are seated at a round table during a wedding dinner. Find the number of ways that everyone can be seated, such that Mr and Mrs Lee are seated together, but their children are not all seated together. [3]

10 A company manufactures packets of potato chips with X mg of sodium in each packet. It is known that the mean amount of sodium per packet is 1053 mg. After some alterations to the production workflow, 50 randomly chosen packets of potato chips were selected for analysis. The amount of sodium in each packet was measured, and the results are summarised as follows:

$$\sum(x-1050)=58.0,\ \sum(x-1050)^2=2326$$

(i) Test at 5% level of significance whether the mean amount of sodium in a packet of potato chips has changed, after the alterations to the workflow. [6]

(ii) Explain what '5% level of significance' means in this context. [1]

(iii) A second tester conducted the same test at $\alpha\%$ level of significance, for some integer α. However, he came to a different conclusion from the first tester. What is the range of α for which the second tester could have taken? [1]

(iv) Without performing another hypothesis test, explain whether the conclusion in part **(i)** would be different if the alternative hypothesis was that the mean amount of sodium had decreased after the alterations to the workflow. [1]

It is given instead that the standard deviation of amount of sodium in a packet of potato chips is 6.0 mg. The mean amount of sodium of a second randomly chosen sample of 40 packets of potato chips is $\bar{y}$.

(v) A hypothesis test on this sample, at 5% level of significance, led to a conclusion that the amount of sodium has decreased.

Find the range of values of $\bar{y}$, to 1 decimal place.

Explain if it is necessary to make any assumptions about the distribution of the amount of sodium in each packet of potato chips. [3]

11 **(a)** Comment on the following statement: "The product moment correlation coefficient between the amount of red wine intake and the risk of heart disease is approximately -1. Thus we can conclude that red wine intake decreases the risk of heart disease." [1]

(b) During an experiment, the radiation intensity, I, from a source at time t, in appropriate units, is measured and the results are tabulated below.

t	0.2	0.4	0.6	0.8	1.0
I	2.81	1.64	0.93	0.55	0.30

(i) Identify the independent variable and explain why it is independent. [1]

(ii) Draw a scatter diagram of these data. With the help of your diagram, explain whether the relationship between I and t is likely to be well modelled by an equation of the form $I = at + b$, where a and b are constants. [3]

(iii) Calculate, to 4 decimal places, the product moment correlation coefficient between

(a) I and t,

(b) $\ln I$ and t. [2]

(iv) Using the model $I = ae^{bt}$, find the equation of a suitable regression line, and calculate the values of a and b. [3]

(v) Use the regression line found in **(iv)** to estimate the radiation intensity when $t = 0.7$. Comment on the reliability of your estimate. [2]

End of Paper