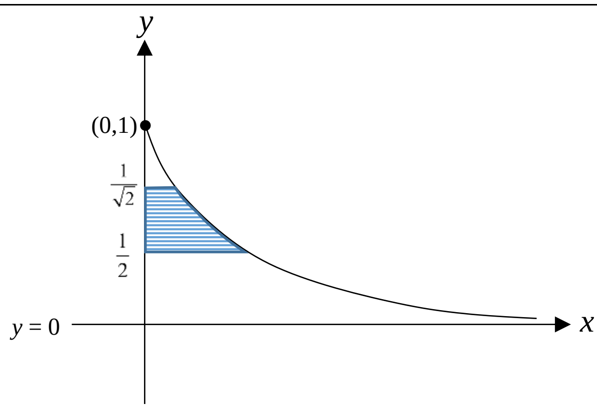
HCI C2 H2 Mathematics Preliminary Examinations 2022 Mark Scheme (Paper 2)

Qn	Suggested Solutions
1	$y=-\left(x-\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2-4$ $=-\left(x-\frac{3}{2}\right)^2-\frac{7}{4}$ 1. Translate $\frac{3}{2}$ units in the positive x direction. 2. Reflect about the x axis 3. Translate $\frac{7}{4}$ units in the negative y direction
	1. Translate $\frac{3}{2}$ units in the positive x direction. 2. Translate $\frac{7}{4}$ units in the positive y direction 3. Reflect about the x axis
2(i)	
(ii)	$\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} x\,\mathrm{d}y$ $=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \sin t\tan t(-\sin t)\,\mathrm{d}t$ $=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} -\sin^2 t\tan t\,\mathrm{d}t$ $=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\cos^2 t-1\right)\tan t\,\mathrm{d}t$ $\frac{\mathrm{d}y}{\mathrm{d}t}=-\sin t$ when $y=\frac{1}{2}$, $t=\frac{\pi}{3}$ when $y=\frac{1}{\sqrt{2}}$, $t=\frac{\pi}{4}$

Qn	Suggested Solutions
	$=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \sin t\cos t-\tan t \, \mathrm{d}t \quad \left(=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin 2t}{2}-\tan t \, \mathrm{d}t\right)$ $=\left[\frac{\sin^2 t}{2}+\ln(\cos t)\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}$ or $\left[-\frac{1}{4}\cos 2t+\ln(\cos t)\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}$ $=\left[\frac{1}{4}+\ln\left(\frac{1}{\sqrt{2}}\right)-\frac{3}{8}-\ln\left(\frac{1}{2}\right)\right]$ $=\ln\sqrt{2}-\frac{1}{8}$ units^2
3(i)	<u>**Method 1**</u> $z=2(\cos\beta+\mathrm{i}\sin\beta)=2\mathrm{e}^{\mathrm{i}\beta}$ $\frac{z}{4-z^2}=\frac{2\mathrm{e}^{\mathrm{i}\beta}}{4-4\mathrm{e}^{\mathrm{i}(2\beta)}}$ $=\frac{\mathrm{e}^{\mathrm{i}\beta}}{2\mathrm{e}^{\mathrm{i}\beta}\left(\mathrm{e}^{-\mathrm{i}\beta}-\mathrm{e}^{\mathrm{i}\beta}\right)}$ $=-\frac{1}{2\left(\mathrm{e}^{\mathrm{i}\beta}-\mathrm{e}^{-\mathrm{i}\beta}\right)}$ $=-\frac{1}{2(2\sin\beta)\mathrm{i}} \quad \left(\because \mathrm{e}^{\mathrm{i}\beta}-\mathrm{e}^{-\mathrm{i}\beta}=2\,\mathrm{Im}\left(\mathrm{e}^{\mathrm{i}\beta}\right)\mathrm{i}\right)$ $=\left(\frac{1}{4}\mathrm{cosec}\beta\right)\mathrm{i}$ where $k=\frac{1}{4}$ $\therefore k=\frac{1}{4}$
	<u>**Method 2**</u> $\frac{z}{4-z^2}=\frac{2\cos\beta+\mathrm{i}(2\sin\beta)}{4-4\cos^2\beta-\mathrm{i}(8\sin\beta\cos\beta)+4\sin^2\beta}$ $=\frac{2\cos\beta+\mathrm{i}(2\sin\beta)}{4-4(1-\sin^2\beta)-\mathrm{i}(8\sin\beta\cos\beta)+4\sin^2\beta}$ $=\frac{2\cos\beta+\mathrm{i}(2\sin\beta)}{8\sin^2\beta-\mathrm{i}(8\sin\beta\cos\beta)}$ $=\frac{\mathrm{i}(2\sin\beta-2\cos\beta\mathrm{i})}{4\sin\beta(2\sin\beta-2\cos\beta\mathrm{i})}$ $=\left(\frac{1}{4}\mathrm{cosec}\beta\right)\mathrm{i}$ where $k=\frac{1}{4}$

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$\arg(w)=\arg\left(-\sqrt{3}+\mathrm{i}\right)=\frac{5\pi}{6}$
$\Rightarrow \arg\left(w^*\right)=-\frac{5\pi}{6}$

$\arg\left(\frac{z}{4-z^2}\right)=\arg\left[\left(\frac{1}{4}\operatorname{cosec}\beta\right)\mathrm{i}\right]=\frac{\pi}{2}$, since for $0<\beta<\frac{\pi}{2}$, $\frac{1}{4}\operatorname{cosec}\beta>0$.

$\arg\left(\left(\frac{z}{4-z^2}\right)\left(w^*\right)^n\right)=\arg\left(\frac{z}{4-z^2}\right)+\arg\left(\left(w^*\right)^n\right)$
$=\arg\left(\frac{z}{4-z^2}\right)+n\arg\left(w^*\right)$
$=\frac{\pi}{2}+n\left(-\frac{5\pi}{6}\right)$
$=\frac{\pi}{2}-\frac{5n\pi}{6}$

For $\left(\frac{z}{4-z^2}\right)\left(w^*\right)^n$ to be a real number, $\arg\left(\left(\frac{z}{4-z^2}\right)\left(w^*\right)^n\right)=k\pi$, where k is an integer.

Therefore
$\frac{\pi}{2}-\frac{5n\pi}{6}=k\pi$
$\Rightarrow n=\frac{3-6k}{5}$
Hence using GC, the three smallest positive integers are

$n=3$ (when $k=-2$),
$n=9$ (when $k=-7$),
and $n=15$ (when $k=-12$).

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Method 2:
$\arg\left(w^*\right)^n=n\arg\left(w^*\right)$
$=n\left(-\frac{5\pi}{6}\right)$
For $\left(\frac{z}{4-z^2}\right)\left(w^*\right)^n$ to be a real number, $\arg\left(w^*\right)^n=\frac{\pi}{2}+k\pi$, where k is an integer.

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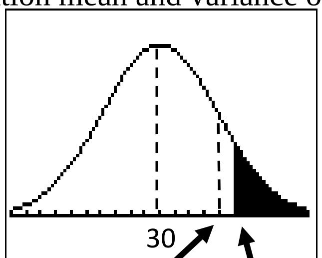
Qn	Suggested Solutions
	$-\frac{5n\pi}{6}=\frac{\pi}{2}+k\pi$ $n=-\frac{3}{5}-\frac{6k}{5}$ using GC, the three smallest positive integers are $n=3$ (when $k=-3$), $n=9$ (when $k=-8$), and $n=15$ (when $k=-13$).
4(a) (i)	$\sum_{r=1}^{k}\left[\left(-\frac{1}{2}\right)^{r+1}+\ln(r+1)\right]$ $=\sum_{r=1}^{k}\left(-\frac{1}{2}\right)^{r+1}+\sum_{r=1}^{k}\ln(r+1)$ $=\sum_{r=1}^{k}\left[\left(-\frac{1}{2}\right)^{r+1}\right]+\ln 2+\ln 3+\ln 4+...+\ln(k+1)$ $=\left(\frac{1}{4}\right)\left[\frac{1-\left(-\frac{1}{2}\right)^{k}}{1-\left(-\frac{1}{2}\right)}\right]+\ln\left[(k+1)!\right]$ $=\frac{1}{6}\left[1-\left(-\frac{1}{2}\right)^{k}\right]+\ln\left[(k+1)!\right]$
4 (a) (ii)	As $k\to\infty$, $\lim_{k\to\infty}\left\{\frac{1}{6}\left[1-\left(-\frac{1}{2}\right)^{k}\right]\right\}=\frac{1}{6}$ $\lim_{k\to\infty}\left\{\ln\left[(k+1)!\right]\right\}=\infty$ Therefore, the sum to infinity of the series does not exist.
(b)	Let *a* be the first term of AP and *d* be the common difference. $S_6=4.5$ $\Rightarrow\frac{6}{2}(2a+5d)=4.5$ $\Rightarrow 2a+5d=1.5$

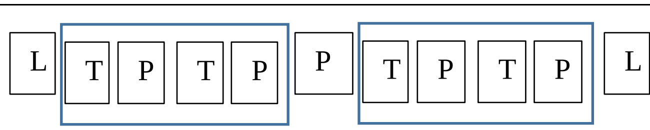
Qn	Suggested Solutions
	$u_1u_2u_3u_4=0$ $\Rightarrow a(a+d)(a+2d)(a+3d)=0$ $\therefore d=-a$ or $-\frac{a}{2}$ or $-\frac{a}{3}$ When $d=-a$, $\Rightarrow 2a+5(-a)=1.5$ $\Rightarrow a=-\frac{1}{2}$ (rej. $\because a>0$) When $d=-\frac{a}{2}$, $\Rightarrow 2a+5\left(\frac{-a}{2}\right)=1.5$ $\Rightarrow a=-3$ (rej. $\because a>0$) When $d=-\frac{a}{3}$, $\Rightarrow 2a+5\left(\frac{-a}{3}\right)=1.5$ $\Rightarrow a=4.5$ $\therefore T_{13}=4.5+12(-1.5)=-13.5$
5(i)	$s=12$
5(ii)	Given l_{DM} is parallel to $\begin{pmatrix}1\\1\\t\end{pmatrix}$ Plane *ABFE* : $6x-z=36\Rightarrow \underset{\sim}{r}\bullet\begin{pmatrix}6\\0\\-1\end{pmatrix}=36$ If *DM* doesn't intersect with *ABFE*, then *DM* must be parallel to *ABFE*, i.e. perpendicular to the normal vector of *ABFE*. $\begin{pmatrix}1\\1\\t\end{pmatrix}\bullet\begin{pmatrix}6\\0\\-1\end{pmatrix}=0\Rightarrow 6-t=0$ $\therefore t=6$
(iii)	From part (i), $t=6$,

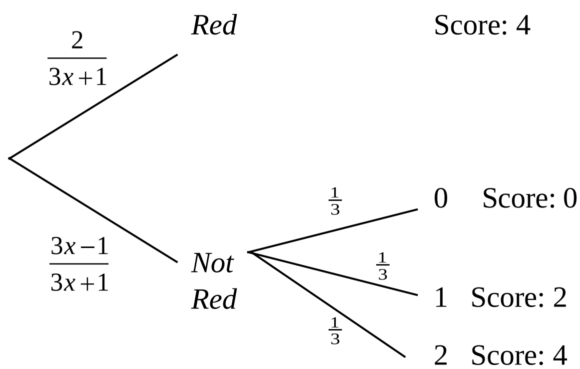
Qn	Suggested Solutions
	$\overrightarrow{DM}$ is parallel to $\begin{pmatrix}1\\1\\6\end{pmatrix}$ $\overrightarrow{DG} // \begin{pmatrix}0\\1\\0\end{pmatrix}$ Normal $= \begin{pmatrix}1\\1\\6\end{pmatrix} \times \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}-6\\0\\1\end{pmatrix} = -\begin{pmatrix}6\\0\\-1\end{pmatrix}$ Equation of plane: $\underset{\sim}{r} \bullet \begin{pmatrix}6\\0\\-1\end{pmatrix} = \begin{pmatrix}-2\\0\\12\end{pmatrix} \bullet \begin{pmatrix}6\\0\\-1\end{pmatrix} = -24$ (shown) $\therefore k = -24$
(iv)	Normal vector of plane $DGM = \begin{pmatrix}6\\0\\-1\end{pmatrix}$ Normal vector of plane $DEFG = \begin{pmatrix}0\\0\\1\end{pmatrix}$ Angle between the 2 planes DGM and $DENM = \cos^{-1}\left\lvert \dfrac{\begin{pmatrix}6\\0\\-1\end{pmatrix} \cdot \begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{37}} \right\rvert$ $= 80.53768° = 80.5°$ (1 d.p.)
(v)	$\overrightarrow{OM} = \begin{pmatrix}-2+\lambda\\ \lambda\\ 12+6\lambda\end{pmatrix}$ Since height of the structure is 26 units, $12 + 6\lambda = 24$ $\lambda = 2$ $\overrightarrow{OM} = \begin{pmatrix}0\\2\\24\end{pmatrix}$

Qn	Suggested Solutions
	Coordinates of $M(0,\ 2,\ 24)$
	$\overrightarrow{AM}=\begin{pmatrix}0\\2\\24\end{pmatrix}-\begin{pmatrix}6\\0\\0\end{pmatrix}=\begin{pmatrix}-6\\2\\24\end{pmatrix}$ Shortest distance $=\dfrac{\left\lvert\overrightarrow{AM}\bullet\begin{pmatrix}6\\0\\-1\end{pmatrix}\right\rvert}{\sqrt{37}}$ $=\dfrac{\left\lvert\begin{pmatrix}-6\\2\\24\end{pmatrix}\bullet\begin{pmatrix}6\\0\\-1\end{pmatrix}\right\rvert}{\sqrt{37}}=\dfrac{\lvert-36-24\rvert}{\sqrt{37}}=\dfrac{60}{\sqrt{37}}$ Alternative Method Note that plane *AGM* parallel to plane *ABFE* plane AGM : $\underset{\sim}{r}\bullet\dfrac{\begin{pmatrix}6\\0\\-1\end{pmatrix}}{\sqrt{37}}=\dfrac{-26}{\sqrt{37}}$ plane $ABFE$: $\underset{\sim}{r}\bullet\dfrac{\begin{pmatrix}6\\0\\-1\end{pmatrix}}{\sqrt{37}}=\dfrac{36}{\sqrt{37}}$ Distance between the 2 planes = $\dfrac{36-(-24)}{\sqrt{37}}=\dfrac{60}{\sqrt{37}}$ units
(vi)	$\prod\ :\ x=c\Rightarrow\underset{\sim}{r}\ \bullet\begin{pmatrix}1\\0\\0\end{pmatrix}=c$ which is a *y*-*z* plane $D(-2,\ 0,\ 12)$ Since DE =10 units $D(8,\ 0,\ 12)$ Since they intersect at E, $x=c=8$
6(a)	Let *T* and *S* denote the event that a triangle and square block is chosen respectively. Let *C* denote the event that a block is correctly placed into the shaper sorter toy.

Qn	Suggested Solutions
	<u>Method 1</u> $\mathrm{P}(C)=0.9$ $\mathrm{P}(S\mid C)=0.4 \Rightarrow \mathrm{P}(T\mid C)=0.6$ $\Rightarrow \dfrac{\mathrm{P}(T\cap C)}{\mathrm{P}(C)}=0.6$ $\Rightarrow \mathrm{P}(T\cap C)=(0.6)(0.9)=0.54$ Required probability $=1-0.54=0.46$ **<u>Method 2</u>** $\mathrm{P}(C)=0.9$ $\Rightarrow \mathrm{P}(C')=0.1$ $\mathrm{P}(S\mid C)=0.4$ $\Rightarrow \dfrac{\mathrm{P}(S\cap C)}{\mathrm{P}(C)}=0.4$ $\Rightarrow \mathrm{P}(S\cap C)=(0.4)(0.9)=0.36$ Required probability $=\mathrm{P}(C')+\mathrm{P}(S\cap C)$ $=0.1+0.36=0.46$
(b)	$\mathrm{P}(S=2)=2\mathrm{P}(S=4)$ $\dfrac{{}^{r+1}C_2\,{}^5C_3}{{}^{r+6}C_5}=2\left(\dfrac{{}^{r+1}C_4\,{}^5C_1}{{}^{r+6}C_5}\right)$ $\dfrac{(r+1)!(10)}{(r-1)!2!}=2\dfrac{(r+1)!(5)}{(r-3)!4!}$ $\dfrac{1}{2(r-1)!}=\dfrac{1}{(r-3)!4!}$ $12(r-3)!=(r-1)!$ $12(r-3)!=(r-3)!(r-2)(r-1)$ $12=(r-2)(r-1)$ since $r\ge 3$ $r^2-3r-10=0$ $(r+2)(r-5)=0$ $r=-2$ (rej) or $r=5$
7(i)	It is **not necessary** for the **fat content of the chocolate bars** to be **normally distributed**. As the sample size (number of chocolate bars = 40) used is large, by **Central Limit Theorem**, the **sample mean fat content** of the chocolate bars is **approximately normally distributed** for the test to be valid.

Qn	Suggested Solutions
(ii)	$\bar{x} = \frac{1220}{40} = 30.5$ $s^2 = \frac{1}{n-1}\sum\left(x-\bar{x}\right)^2$ $= \frac{50}{39}$ ≈ 1.28205 ≈ 1.28 (3 s.f.)
(iii)	Let X be the fat content in a chocolate bar in g. Let μ and σ be the population mean and variance of X. $\text{H}_0 : \mu = 30$ $\text{H}_1 : \mu > 30$  Under H_0, since n is large, by Central Limit Theorem, $\overline{X} \sim \text{N}\left(\mu, \frac{\sigma^2}{n}\right)$ approximately. Test statistic, $Z = \frac{\overline{X}-\mu}{S/\sqrt{n}} \sim \text{N}(0,1)$ approximately. For test to be rejected at 5% level of significance, $z = \frac{\bar{x}-\mu}{s/\sqrt{n}} \geq 1.64485$ $\frac{\bar{x}-30}{\sqrt{\frac{50}{(39)(40)}}} \geq 1.64485$ $\bar{x} \geq 30.294475$ $\bar{x} \geq 30.3$ (3 s.f.) Since $\bar{x} = \frac{1220}{40} = 30.5 \geq 30.2945$, we reject H_0 at the 5% level of significance and conclude that the manager's suspicion is valid.
(iv)	From part **(iii)**, it was concluded that the manager's suspicion is valid, i.e. reject H_0 $\Rightarrow$ Test statistic is in the critical region . $\Rightarrow$ Test statistic ≥ 1.64486

Qn	Suggested Solutions
	Now with a smaller population variance, the new test statistic will be larger. $\Rightarrow$ New Test statistic value >Old Test statistic value i.e $\dfrac{\bar{x}-\mu}{\sqrt{\dfrac{\sigma^2}{n}}} > \dfrac{\bar{x}-\mu}{\sqrt{\dfrac{s^2}{n}}}$ H_0 is still rejected at 5% level of significance. Thus the conclusion will be the same, i.e. conclude that the manager's suspicion is valid.
8(a)	<u>**Method 1**</u> Total number of committees formed $=\binom{5}{2}\times\binom{10}{4}\times\binom{8}{4}$ $=147000$ Number of committees with the couple serve together $=\binom{5}{2}\times\binom{8}{2}\times\binom{8}{4}$ $=19600$ Required number of committees formed $=147000-19600$ $=127400$
	<u>**Method 2**</u> <u>Case 1</u>: Wife is in and husband is out No. of committees $=\binom{5}{2}\times\binom{8}{3}\times\binom{8}{4}=39200$ <u>Case 2</u>: Wife us out and husband is in No. of committees $=\binom{5}{2}\times\binom{8}{3}\times\binom{8}{4}=39200$ <u>Case 3</u>: The couple is out No. of committees $=\binom{5}{2}\times\binom{8}{4}\times\binom{8}{4}=49000$ Required number of committees formed = 39200 + 39200 + 49000 = 127400
(b)	 Number of arrangements if no two parents are to stand next to each other $=2!\times4!\times4!\times3\times3$ =10368
(c)	No. of circular arrangements if all parents are together and teachers are separated

Qn	Suggested Solutions
	$=3!\times 4!\times 4!$ $=3456$ Required probability $=\dfrac{3!4!4!}{10!}=\dfrac{1}{1050}$ $=9.52\times 10^{-4}$ (3 s.f.)
9(i)	Let S be the score of Abel's game. $\mathrm{P}(S=4)$ $=\mathrm{P}(\{\text{Red}\},\{\text{Non-red, black 2}\})$ $=\dfrac{2}{3x+1}+\dfrac{3x-1}{3x+1}.\dfrac{1}{3}$ $=\dfrac{6+3x-1}{3(3x+1)}$ $=\dfrac{3x+5}{3(3x+1)}$ 
(ii)	Let S be Abel's score in a round. $\mathrm{P}(S=0)$ $=\mathrm{P}(\{\text{Non-red, Card zero}\})$ $=\dfrac{3x-1}{3x+1}.\dfrac{1}{3}$ $=\dfrac{3x-1}{3(3x+1)}$ $\mathrm{P}(S=2)$ $=\mathrm{P}(\{\text{Non-red, Card 1}\})$

<table>
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s	0	2	4
$\mathrm{P}(S=s)$	$\dfrac{3x-1}{3(3x+1)}$	$\dfrac{3x-1}{3(3x+1)}$	$\dfrac{3x+5}{3(3x+1)}$

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<tr><td>(iii)</td><td>Mode = 4</td></tr>
<tr><td>(iv)</td><td>

$\mathrm{E}(S)$

$= 0\cdot\mathrm{P}(S=0)+2\cdot\mathrm{P}(S=2)+4\mathrm{P}(S=4)$

$= 2\left(\dfrac{3x-1}{3(3x+1)}\right)+4\left(\dfrac{3x+5}{3(3x+1)}\right)$

$= \dfrac{6(x+1)}{(3x+1)}$

$\mathrm{E}(S^2)$

$=0^2\cdot\mathrm{P}(S=0)+2^2\cdot\mathrm{P}(S=2)+4^2\cdot\mathrm{P}(S=4)$

$= 2^2\left(\dfrac{3x-1}{3(3x+1)}\right)+4^2\left(\dfrac{3x+5}{3(3x+1)}\right)$

$= \dfrac{12x-4+48x+80}{3(3x+1)}$

$= \dfrac{60x+76}{3(3x+1)}$

$\therefore\ \mathrm{Var}(S)=\mathrm{E}(S^2)-[\mathrm{E}(S)]^2$

$= \dfrac{60x+76}{3(3x+1)}-\dfrac{36(x+1)^2}{(3x+1)^2}$

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$x=3$

$\mathrm{E}(S)=\dfrac{18(3)+18}{3(10)}=2.4$

$\mathrm{Var}(S)=\mathrm{E}(S^2)-[\mathrm{E}(S)]^2$

$=\dfrac{60(3)+76}{3(3(3)+1)}-(2.4)^2$

$=\dfrac{256}{3(10)}-(2.4)^2=\dfrac{208}{75}$

$n=100$ is large, by Cental Limit Theorem,

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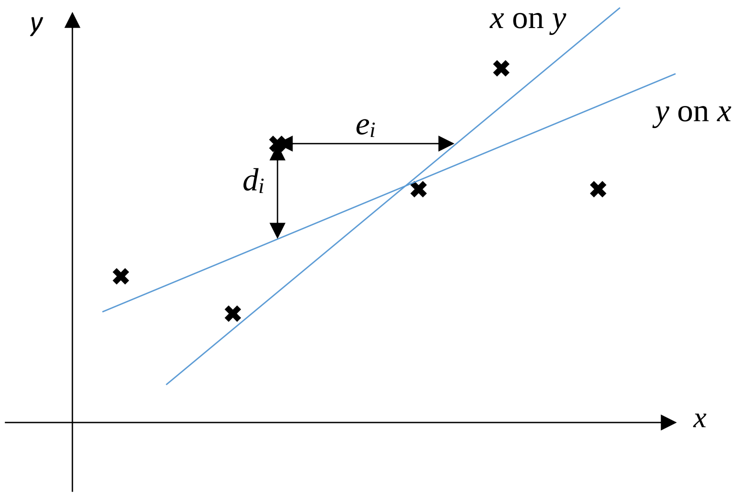
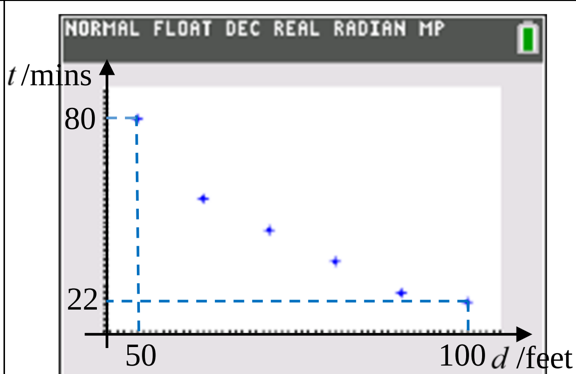
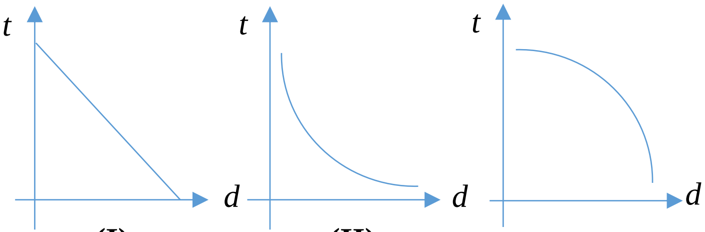
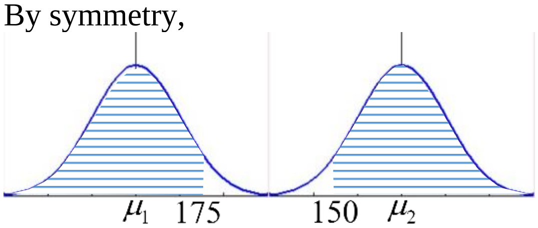
Qn	Suggested Solutions
	$\bar{S} \sim \mathrm{N}\left(2.4, \dfrac{208}{75(100)}\right)$ approximately. $\mathrm{P}\left(\bar{S} \geq 2.5\right) = 0.2740929 = 0.274$ (3 s.f.)
10(a)	 The regression line of y on x is the line which minimizes the sum of squares of the residuals in the vertical direction, d_i, i.e. $\sum d_i^2$, while the regression line of x on y is the line which minimizes the sum of squares of the residuals in the horizontal direction, e_i, i.e. $\sum e_i^2$
(b)(i)	

Qn	Suggested Solutions
(ii)	The scatter diagram shows a decreasing, concave upward trend. Hence model (II) is a better fit for the data.  **(I)** $t = ad + b$ **(II)** $t = a\left(\frac{1}{d}\right) + b$ **(III)** $t = ae^{d} + b$ From GC, $t = 5767.446342\left(\frac{1}{d}\right) - 37.61923382$ $\therefore t = 5767.446\left(\frac{1}{d}\right) - 37.619 \text{ (3 d.p.)}$ $r = 0.995 \text{ (3 d.p.)}$
	(I) Decreasing, linear relationship **(II)** Non-linear, Decreasing and concave upwards trend **(III)** Non-linear, Decreasing and concavce downwards trend Since scatter diagram in **(b)(i)** shows a non-linear relationship between the 2 variables and the characteristics of the scatter plot shows that a decreasing and concave upwards curve like in Model (II) will best model the relationship for the 2 variables.
(iii)	When $d = 150$, $t = 5767.446342\left(\frac{1}{150}\right) - 37.61923382$ $t = 0.830 \text{ (3 s.f.)}$ The estimate is not reliable because $d = 150$ is outside the data range $[50, 100]$.
(iv)	$1 \text{ m } = 3.28 \text{ ft} \Rightarrow D \text{ m} = 3.28D \text{ ft} = d$ $\therefore d = 3.28D$ $t = 5767.446342\left(\frac{1}{3.28D}\right) - 37.61923382$ $t = \frac{1760}{D} - 37.6 \text{ (3 s.f.)}$
(v)	From GC, $\left(\overline{\frac{1}{d}}, \bar{t}\right) = (0.0141, 43.7) \text{ (3 s.f.)}$

Qn	Suggested Solutions
	$\overline{\frac{1}{d}}=0.0140939153$ $\bar{d}=\frac{1}{0.0140939153}$ $\bar{d}=70.9526=71.0$ (3 s.f.)
11	$X\sim \text{N}(\mu_1,11.83^2),\ Y\sim \text{N}(\mu_2,11.83^2)$ Given that $\text{P}(X<175)=\text{P}(Y>150)$, $\text{P}\left(Z<\frac{175-\mu_1}{11.83}\right)=\text{P}\left(Z>\frac{150-\mu_2}{11.83}\right)$ $\frac{175-\mu_1}{11.83}=-\frac{150-\mu_2}{11.83}$ $175-\mu_1=-150+\mu_2$ $\therefore \mu_1+\mu_2=175+150$ $=325$
	By symmetry,  $175-\mu_1=\mu_2-150$ $\therefore \mu_1+\mu_2=175+150$ $=325$
(i)	$X_1+X_2-2(Y+M)\sim \text{N}(2(180)-2(180),\ 2(11.83^2)+2^2(174.9953))$ i.e. $X_1+X_2-2(Y+M)\sim \text{N}(0,\ 979.879)$ $\text{P}(0<X_1+X_2-2Y<15)=0.184$ (3 s.f.) Assume that the volumes of each cup of black coffee and milk added from the vending machine are (i.e. X_1 , X_2 , Y and M) independent of one another.
(ii)	$X\sim \text{N}(180,11.83^2),\ Y\sim \text{N}(145,11.83^2), M\sim \text{N}(35,5.92^2)$ B = Cost Price of 1 cup of Black Coffee = $0.01X$ W = Cost Price of 1 cup of White Coffee = $0.01Y+0.02M$ $B\sim \text{N}(1.8, 0.1399489)$, $W=0.01\text{Y}+0.02\text{M}\sim \text{N}(2.15, 0.02801345)$ Since n cups of black coffee are sold per day, $(100-n)$ cups of white coffee are sold per day.

Qn	Suggested Solutions

Let P_B be profit for black coffee, and P_W be profit for white coffee and T be the total profit per day.

$P_B = 4n - (B_1 + B_2 + ... + B_n)$

$\mathrm{E}(P_B) = 2.2n$

$\mathrm{Var}(P_B) = 0.01^2 (11.83^2) n$

$= 0.01399489n$

$\mathrm{E}(P_W) = 5(100-n) - 2.15(100-n) = 285 - 2.85n$

$\mathrm{Var}(P_W) = (100-n)0.02801345$

$T \sim \mathrm{N}(285 - 0.65n,\ 2.801345 - 0.01401856n)$

$\mathrm{P}(T > 230) \geq 0.8$

n	$\mathrm{P}(T > 230)$
81	0.9657
82	0.907
83	0.794

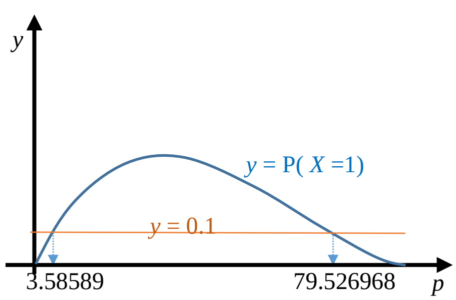
Therefore, largest number of cups of black coffee sold per day is 82.

(iii) Let F be the number of customers selecting regular black coffee receives the drink free of charge.

$F \sim \mathrm{B}\left(3, \frac{p}{100}\right)$

$\mathrm{P}(F=1) = 3\left(\frac{p}{100}\right)\left(1-\frac{p}{100}\right)^2$

(iv) $3\left(\frac{p}{100}\right)\left(1-\frac{p}{100}\right)^2 \leq 0.1$



$0 \leq p \leq 3.58$ or $79.6 \leq p \leq 100$ (3 s.f.)