**RIVER VALLEY HIGH SCHOOL**
**2019 JC2 Preliminary Examination**
Higher 2

| NAME | |
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| CLASS | | | | | |
|---|---|---|---|---|---|

| INDEX NUMBER | | |
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# MATHEMATICS 9758/02

Paper 2

**23 September 2019**

**3 hours**

Candidates answer on the Question Paper
Additional Materials: List of Formulae (MF26)

### READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
You are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

| **For examiner's use only** | |
|---|---|
| Question number | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| | |
| **Total** | |

| **Calculator Model:** |
|---|
| |

This document consists of **7** printed pages.

## Section A: Pure Mathematics [40 marks]

1 A sequence $u_0$, $u_1$, $u_2$, ... is such that $u_{n+1} = -3u_n + An + B$, where $A$ and $B$ are constants and $n \geq 0$.

(i) Given that $u_0 = 4$, $u_1 = 2$ and $u_2 = 16$, find the values of $A$ and $B$. [3]

It is known the $n$th term of the same sequence is given by $u_n = a(-3)^n + bn + c$, where $a$, $b$ and $c$ are constants.

(ii) By considering a system of linear equations, find the values of $a$, $b$ and $c$. [3]

2 Two planes $p_1$ and $p_2$ have equations

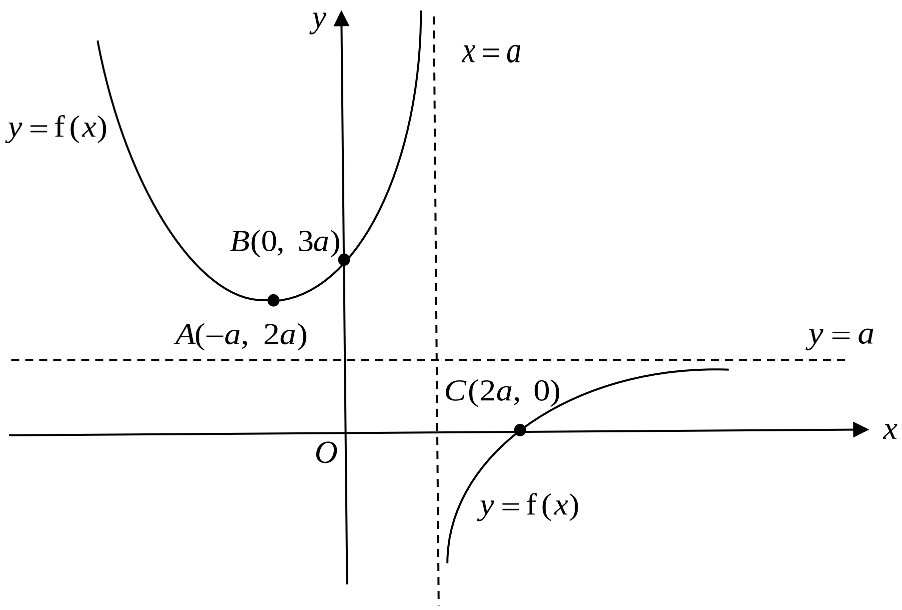
$$p_1 : \mathbf{r} = \mathbf{a} + \lambda_1\mathbf{m} + \mu_1\mathbf{n},$$
$$p_2 : \mathbf{r} = \mathbf{b} + \lambda_2\mathbf{m} + \mu_2\mathbf{n},$$

where $\lambda_1, \lambda_2, \mu_1, \mu_2 \in \mathbb{R}$.

(a) The points $A$ and $B$ have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The angle between $\mathbf{a}$ and $\mathbf{m}\times\mathbf{n}$ is 45° and that between $\mathbf{b}$ and $\mathbf{m}\times\mathbf{n}$ is 60°. Show that the distance between $p_1$ and $p_2$ is given by $\left|\dfrac{|\mathbf{b}|}{2} - \dfrac{|\mathbf{a}|}{\sqrt{2}}\right|$. [2]

(b) $Q$ is a variable point between $p_1$ and $p_2$ such that the distance of $Q$ from $p_1$ is twice that of the distance from $p_2$. Write down a possible position vector of $Q$, in terms of $\mathbf{a}$ and $\mathbf{b}$. Describe the locus of $Q$ and write down its vector equation. [3]

3 (a) The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph has a minimum point at $A(-a,\ 2a)$ where $a > 0$. It passes through the points $B(0,\ 3a)$ and $C(2a,\ 0)$. The equations of the asymptotes are $x = a$ and $y = a$. Each of the gradients of the tangents at points $B$ and $C$ is $a$.



Sketch on separate clearly labelled diagrams, the graphs of

(i) $y = \mathrm{f}(2x + a)$, [2]

(ii) $y = \mathrm{f}'(x)$, [2]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$. [3]

Label in each case, the coordinates of points corresponding to the points *A*, *B* and *C* (if any) of $y = \mathrm{f}(x)$ and the equation of asymptote(s).

(b) Describe a sequence of transformations which transforms the graph of $2y^2 - x^2 = 1$ onto the graph of $y^2 - (x-1)^2 = 1$. [2]

4 The parametric equations of a curve are

$$x = \sin t, \quad y = \cos t, \quad \text{where } 0 < t < \frac{\pi}{2}.$$

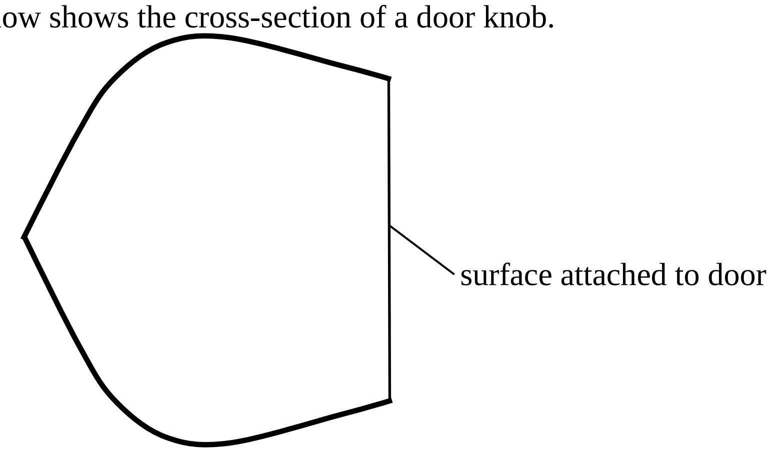
(i) Sketch the graph of the curve. [1]

(ii) Show that the equation of the tangent to the curve at the point where $t = p$ is given by the equation $y = -(\tan p)x + \sec p$. [3]

(iii) Given that the above tangent meets the *x*-axis at the point *P* and the *y*-axis at the point *Q*, find the coordinates of the mid-point *M* of *PQ* in terms of *p*.
Hence, determine the cartesian equation of the locus of *M*. [3]

(iv) Find the exact area of the region enclosed by the curve, the lines $x = \dfrac{1}{2}$, $x = \dfrac{1}{\sqrt{2}}$ and the *x*-axis. [3]

5 The diagram below shows the cross-section of a door knob.

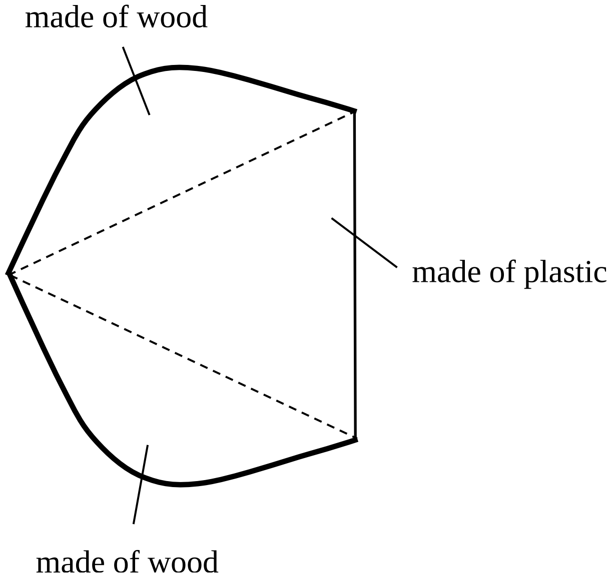



The curve part of the door knob can be modelled by the curve *C* with equation $y = \dfrac{2x}{1+x^2}$ between *x* = 0 and *x* = 2. The solid door knob is obtained by rotating *C* through four right angles about the *x*-axis.

(i) Sketch the graph of *C* for $0 \le x \le 2$. [1]

(ii) Using the substitution $x = \tan\theta$, find the exact volume of the door knob. [6]

The exterior and the interior of the door knob is made of wood and plastic respectively. The interior of the door knob consists of a solid cone.




The cost of wood and plastic used in manufacturing of the door knob is \$3 and \$1.20 per unit volume respectively. Find the cost of manufacturing one such door knob, giving your answer to the nearest dollar. [3]

**Section B: Probability and Statistics [60 marks]**

6 At a funfair game store, 3 red, 2 blue, 1 white, 1 yellow and 1 green beads are being arranged. The beads are identical apart from their colours.

(i) Find the number of ways to arrange all the beads in a row with all the red beads separated. [2]

(ii) Five beads, one of each colour, are string together to form a rigid ring. Find the number of distinct rings that can be formed. [2]

At another game store, the letters of the word INITIATE are being arranged in a row instead. Find the probability that all the letters are used and the letters A, E and N are arranged in alphabetical order. [2]

7 In a game, Alfred is given 6 keys of which $n$ of the keys can unlock a box. He tries the keys randomly without replacement to unlock the box.
The random variable $X$ denotes the number of tries Alfred takes to unlock the box.

(i) Write down an expression for $\mathrm{P}(X=3)$ in terms of $n$. [1]

Use $n$ = 3 for the rest of this question.
For the game, Alfred wins \$2.00 if he takes less than 4 tries to open the box else he wins \$1.00. The random variable $W$ denotes Alfred's winnings in a game.

(ii) Find the probability distribution table for $W$ and hence find the value of $\mathrm{E}(W)$. [2]

(iii) Find the probability that Alfred wins at least \$9.00 after playing the game 5 times. [3]

8 Two boxes, one white and one black, are used in a game. The boxes contains balls labelled '2', '5' and '10'. The balls are identical except for their numbers. The table below shows the number of balls in each box.

| | Number of balls labelled | | |
|---|---|---|---|
| | '2' | '5' | '10' |
| White box | 5 | 2 | 1 |
| Black box | 5 | 1 | 2 |

In the first round of the game, a player draws 3 balls randomly from the white box. If the sum of the numbers on the 3 balls add up to 9 or more, the player enters the second round of the game to draw another 3 balls randomly from the black box.

(i) Find the probability that the player draws 3 different numbers in the first round. [1]

(ii) Show that the probability that sum of the numbers add up to 12 and 6, in the first and second round respectively, is $\frac{25}{1568}$.

Hence, find the probability that the sum of the numbers drawn from the two rounds adds up to 18. [4]

In the third round of the game, only the original black box with its 8 balls is used. The player draws 2 balls from it randomly. Find the expected sum of the numbers drawn. [2]

9 A multiple-choice test consists of 12 questions and each question has 5 options for a candidate to choose from. The candidate has to choose one option as the answer for each question. For every question that a candidature answered correctly, the candidate is awarded 1 mark. There are no marks awarded or deducted for wrong answers. Let $X$ denotes the number of marks a candidate scores for the test.

(i) State an assumption on how a candidate decides which option to choose in order for $X$ to be modelled by a binomial distribution. Explain why this assumption is necessary. [2]

Suppose the assumption stated in part (i) holds.

(ii) Find the most probable number of marks that a candidate will score in the test. [2]

(iii) A candidate found that he will score at most 5 marks. Find the probability that he will score more than 4 marks. [2]

(iv) A candidate sat for $n$ such tests (where $n$ is large), with each test consisting of 12 questions and each question has 5 options. Determine the least value of $n$ so that the probability of obtaining a mean test score of at most 2.7 marks is 0.95. [2]

10 Engineers claimed that a newly developed engine is efficient. The number of hours $x$ the engine ran on one litre of fuel was measured on 70 occasions. The results are summarized by

$$\sum(x-46)=-27, \quad \sum(x-46)^2=30939.$$

(i) Calculate unbiased estimates for the population mean and variance of the number of hours the engine runs on one litre of fuel. [2]

(ii) Determine the greatest mean number of hours the engineers should claim so that there is sufficient evidence at the 5% level of significance that the engine is efficient. [3]

(iii) The engineers collected five more data points:

56, 34, 63, 50, 54.

The engineers found that when all 75 data points are considered, they can no longer claim that the engine is efficient at the $\alpha$ % level of significance.
Show that the unbiased estimate for the population variance, for the set of 75 data points is 426.378, correct to 3 decimal places. Find the greatest integer value of $\alpha$. [4]

11 (a) The equation of the estimated least squares regression line of $y$ on $x$ for a set of bivariate data is $y=a+bx$. Explain what do you understand by the least squares regression line of $y$ on $x$. [2]

(b) A student wishes to determine the relationship between the length of a metal wire $l$ in millimetres, and the duration of time $t$ in minutes for it to completely dissolve in a particular chemical solution. After conducting the experiment, he obtained the following set of data.

| $l$ | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 |
|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0.779 | 1.10 | 1.35 | 1.56 | 1.74 | 1.91 | 2.07 | 2.22 | 2.31 | 2.45 |

(i) Draw a scatter diagram to illustrate the data. [1]

(ii) State with a reason, which of the following model is appropriate for the data collected.

(A) $t=a+bl^2$, (B) $t=a+b\ln l$, (C) $t=ae^{bl}$,

where $a$ and $b$ are some constants. [2]

(iii) For the above chosen model, calculate the values of $a$ and $b$ and the product moment correlation coefficient. [2]

(iv) Using the model in part (iii), estimate the length of metal wire that took 1.00 minute to dissolve completely in the chemical solution.
Comment on the reliability of your answer. [3]

(v) Given that 1 millimetre = 0.03937 inch, find an equation that can be used to estimate the duration of time taken to completely dissolve a metal wire of length $L$, where $L$ is measured in inches. [2]

12 An orchard produces apples and pears.

(i) The masses of apples produced by the orchard have mean 70 grams and standard deviation 40 grams.

Explain why the masses of apples in the orchard are unlikely to be normally distributed. [1]

(ii) The masses of pears produced by the orchard are normally distributed. It is known that one-third of the pears produced weigh less than 148 grams, and one-third weigh more than 230 grams. Find the mean and standard deviation of the masses of pears produced in the orchard. [4]

During each production period, the orchard produces 300 apples and 400 pears and the owner is able to sell all the apples at \$0.005 per gram and all the pears at \$0.008 per gram. It is further known that the owner needs to spend a fixed amount of \$$c$ as the operating cost for each production period and that the orchard makes a profit 90% of the time.

(iii) Explain why the total mass of the 300 apples is approximately normally distributed. [1]

(iv) By letting $R$ be the amount of money collected from selling the 300 apples and 400 pears, find the distribution of $R$ and hence, determine the value of $c$. [6]

**END OF PAPER**