RAFFLES INSTITUTION
2020 YEAR 6 PRELIMINARY EXAMINATION

CANDIDATE NAME

CLASS 20

MATHEMATICS 9758/02

PAPER 2

3 hours

Candidates answer on the Question Paper
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

FOR EXAMINER'S USE							
Q1	Q2	Q3	Q4	Q5		Sub-Total	Total
/5	/5	/8	/10	/12		/40	
Q6	Q7	Q8	Q9	Q10	Q11	Sub-Total	/100
/7	/8	/9	/12	/12	/12	/60	

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RAFFLES INSTITUTION
Mathematics Department

Section A: Pure Mathematics [40 marks]

1 An arithmetic series has first term $\frac{3}{2}$ and fourth term u_4.

A geometric series also has first term $\frac{3}{2}$ and fourth term u_4.

Given that the common ratio of the geometric series is non-negative and the sum of its first 3 terms is $\frac{21}{2}$, find the sum of the first 10 odd-numbered terms of the arithmetic series. [5]

2 In a geometric series, the ratio of the sum of the first 8 terms to the sum of the first 4 terms is 17:16. Find the 2 possible values of the common ratio, r_1, r_2 where $r_1 > r_2$. The sum to infinity of the geometric series with first term *a* and common ratio r_1 is denoted by S_1 and the sum to infinity of the geometric series with first term *b* and common ratio r_2 is denoted by S_2. Find $S_1 : S_2$ in terms of *a* and *b*. [5]

3 The functions f and g are defined by

$$\mathrm{f}(x) = 1 + 3\mathrm{e}^{-x}, \qquad x \in \mathbb{R}, x > 0,$$
$$\mathrm{g}(x) = |x-1|(x-3), \qquad x \in \mathbb{R}, x < c \text{ where } c \text{ is a real constant.}$$

(i) Given that $c = 4$, determine if the composite functions fg and gf exist, justifying your answers. Find the range of the composite function that exists. [4]

(ii) Given that g^{-1} exists, state the largest possible value of c. Using this value of c, find $\mathrm{g}^{-1}(x)$. [4]

4 Given that $y = \mathrm{e}^{\tan^{-1}\left(\frac{x}{2}\right)}$, show that $\left(4+x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y$. [2]

(i) By repeated differentiation of the above result, find the Maclaurin series for $\mathrm{e}^{\tan^{-1}\left(\frac{x}{2}\right)}$ up to and including the term in x^3. [5]

(ii) Hence find the Maclaurin series for $\dfrac{\mathrm{e}^{\tan^{-1}\left(\frac{x}{2}\right)}}{(1+x)^2}$ up to and including the term in x^2. [3]

5 Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. Point C lies on OA, between O and A, such that $OC:CA = 2:1$. Point D lies on OB produced such that $OD:BD = 3:2$.

(i) Find the position vectors $\overrightarrow{OC}$ and $\overrightarrow{OD}$, giving your answers in terms of **a** and **b**. [2]

(ii) Show that the point E where the lines BC and AD meet has position vector $\dfrac{4}{3}\mathbf{a} - \mathbf{b}$. [4]

(iii) Show that the area of triangle CDE can be written as $k\left|\mathbf{a}\times\mathbf{b}\right|$, where k is a constant to be found. [3]

(iv) It is given that the point F is on BO produced, and OE bisects the angle AOF. Find the ratio $OA:OB$. [3]

Section B: Probability and Statistics [60 marks]

6 For events X and Y, it is given that $\mathrm{P}(X \cap Y') = \frac{1}{2}$, $\mathrm{P}(X \cup Y') = \frac{3}{4}$ and $\mathrm{P}(X | Y') = \frac{50}{63}$. Find

(i) $\mathrm{P}(Y')$, [2]

(ii) $\mathrm{P}(X)$, [2]

(iii) $\mathrm{P}(X \cap Y)$ and state with a reason whether X and Y are independent events. [3]

7 In a factory, machines pack sugar into bags of 1 kg each on average, with variance σ^2 kg^2. The manufacturer is concerned that the machines are putting too much sugar into the bags and decides to carry out a hypothesis test. A random sample of 8 bags are selected and their total mass is 8.4 kg.

(i) Stating a necessary assumption, carry out a test of the manufacturer's concern at the 5% significance level if $\sigma = 0.08$. [5]

(ii) Use an algebraic method to calculate the range of values of σ^2 for which the null hypothesis would not be rejected at the 5% significance level. [3]

8 **In this question you should state clearly the values of the parameters of any normal distribution you use.**

In a supermarket, the masses in grams of apples have the distribution $\mathrm{N}(90, 13^2)$ and the masses in grams of potatoes have the distribution $\mathrm{N}(170, 30^2)$.

(i) Find the probability that the mass of a randomly chosen potato is more than twice the mass of a randomly chosen apple. [3]

A certain salad recipe requires 5 apples and 6 potatoes.

(ii) Find the probability that the total mass of 5 randomly chosen apples and 6 randomly chosen potatoes is between 1.2 and 1.5 kilograms. [3]

The salad recipe requires the apples and potatoes to be prepared by peeling and slicing them. The process reduces the mass of each apple by 15% and the mass of each potato by 25%.

(iii) Find the probability that the total mass, after preparation, of 5 randomly chosen apples and 6 randomly chosen potatoes is not more than 1.2 kilograms. [3]

9 The continuous random variable X has the distribution $\mathrm{N}(\mu, \sigma^2)$.
It is known that $\mathrm{P}(X < k) = 0.2$ and $\mathrm{P}(X < 7) = 0.8$.

(i) Show that $\mathrm{P}(k < X < 7) = 0.6$ and write down the value of $\mathrm{P}(\mu < X < 7)$. [2]

(ii) Express μ in terms of k. [1]

You may use $\sigma^2 = 12$ for the rest of the question.

(iii) Show that $\mu = 4.0845$ correct to 4 decimal places. [3]

(iv) Find $\mathrm{P}(|X| < k)$. [2]

It is given that $2\mathrm{P}(X \le r) = 3\mathrm{P}(X > r)$ for a certain constant, r.

(v) Ten independent observations of X are randomly selected. Find the probability that there are more observations of X with values greater than r than observations of X with values less than r in the selection. [4]

10 A group of 13 people consists of 6 single men and 5 single women and a married couple. A committee of 7 is to be selected from the group.

(i) Find the number of committees that can be formed if there is no restriction in the selection. [1]

(ii) Show that there are 658 such committees with more women than men. [2]

Given that a committee of 7 people is to be selected from the group such that the committee contains more women than men,

(iii) find the probability of getting a committee that consists of 4 single women and 3 single men. [2]

(iv) find the probability of getting a committee that contains at least one married member. [2]

The 4 women and 3 men who were finally selected for the committee included the married couple. The 7 members sit at random around a table with 7 chairs.

(v) Find the probability that the men are all separated from each other. [2]

(vi) Given that the men are all separated from each other, find the probability that the married couple sit next to each other. [3]

11 In conjunction with the Great Singapore Sale, a certain electronics store is having a lucky draw for their customers. In each round of the lucky draw, the customer draws two balls randomly, one after another, with replacement from a box containing 20 red balls, 30 blue balls and 50 white balls. The colour of each ball drawn is noted and points are awarded accordingly as follows.

Colour of ball	**Point(s)**
Red	5
Blue	4
White	1

The customer's score in each round of the lucky draw is the total number of points awarded for the balls drawn. To illustrate, a customer scores a total of 5 points if he draws a blue ball and a white ball in a round of the lucky draw, regardless of the order of appearance of the balls.
You may assume that the 100 balls are indistinguishable from each other apart from their colour.

Let X denote the total number of points scored by a customer in a round of the lucky draw.

(i) Tabulate the probability distribution of X. [3]

(ii) Find $\mathrm{E}(X)$ and show that $\mathrm{Var}(X) = 6.02$. [2]

Mr Lim participated in 50 rounds of the lucky draw.

(iii) Using a suitable approximation, find the probability that Mr Lim's average score is at least 6. [3]

A customer wins a cash voucher if his total score for one round of the lucky draw is more than 6.

The total number of cash vouchers won by Mr Tan in n rounds of the lucky draw is denoted by Y.

(iv) Find the least value of n such that there is a probability of more than 0.7 that Mr Tan will win more than 3 cash vouchers in total. [4]