YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME	

CG		INDEX NO	

MATHEMATICS 9758/01

Paper 1

2 SEPTEMBER 2020

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	1	2	3	4	5	6	7
Marks							

Question	8	9	10	11	Total marks
Marks					/ 100

This document consists of **28** printed pages and **4** blank pages.

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1 Vectors **a**, **b** and **c** are such that $\mathbf{b} \neq \mathbf{0}$ and $3\mathbf{a} \times \mathbf{b} = 7\mathbf{b} \times \mathbf{c}$.

(i) Show that $3\mathbf{a} + 7\mathbf{c} = \lambda \mathbf{b}$, where λ is a constant. [2]

(ii) It is now given that **a** and **b** are unit vectors, that the modulus of **c** is $\frac{3}{7}$ and that the angle between **a** and **c** is 60°. Using a suitable scalar product, find exactly the two possible values of λ. [4]

2 **(i)** Show that $\dfrac{3}{2r+1}-\dfrac{4}{2r+3}+\dfrac{1}{2r+5}=\dfrac{8r+k}{(2r+1)(2r+3)(2r+5)}$, where k is a constant to be found. [2]

(ii) Hence find $\displaystyle\sum_{r=0}^{n} \frac{2r+7}{(2r+1)(2r+3)(2r+5)}$.

(There is no need to express your answer as a single algebraic fraction.) [3]

[Question 2 continues on the next page.]

2 [Continued]

(iii) Explain why $\sum_{r=0}^{\infty} \frac{2r+7}{(2r+1)(2r+3)(2r+5)}$ is a convergent series, and state the value of the sum to infinity. [2]

3 The function f is defined by $\mathrm{f}(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are real numbers.

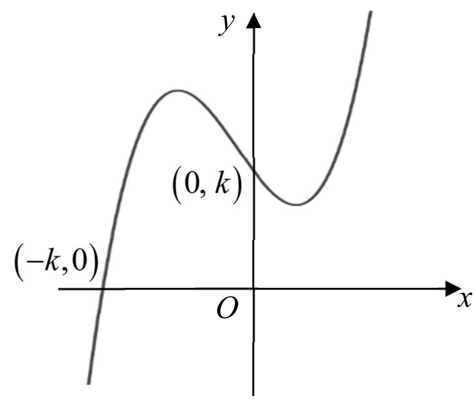
(i) Given that $1-2\mathrm{i}$ and -1 are roots of $\mathrm{f}(x)=0$, find b, c and d in terms of a. [4]

[Question 3 continues on the next page.]

3 **[Continued]**

(ii) Given instead that $a = 2$, $b = -1$ and $c = -4$, find the range of values of d such that the equation $\mathrm{f}(x) = 0$ has only one real root. [2]

(iii) It is now given that the curve with equation $y = \mathrm{f}(x)$ meets the x-axis at $(-k, 0)$ and the y-axis at $(0, k)$, where $k > 0$, as shown in the diagram.



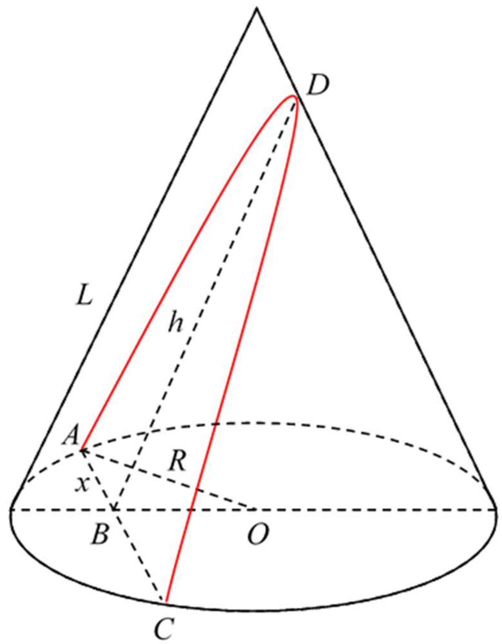
Sketch the graph of $y = \dfrac{1}{\mathrm{f}(x)}$, stating the equations of any asymptotes and the coordinates of any points of intersections with the axes. [2]

4 **(i)** Find the exact roots of the equation $\left|\dfrac{4x-5}{x-2}\right| = 3x$. [3]

(ii) On the same axes, sketch the curves with equations $y = \left|\frac{4x-5}{x-2}\right|$ and $y = 3x$.

Hence solve exactly the inequality $\left|\frac{5-4x}{3x-6}\right| > x$. [5]

5



The diagram shows a parabola formed by the intersection of a right circular cone and a plane parallel to the slanted edge of the cone. The cone with the point O at the centre of its base has radius R cm and slant length L cm, where R and L are constants. The points A, B, C and D on the plane are such that $BD = h$ cm, $AB = BC = x$ cm and the area S of the region bounded by the parabola and the line AC is $\frac{4}{3}xh$.

(i) Show that $S = \dfrac{2Lx\left(R+\sqrt{R^2-x^2}\right)}{3R}$. [2]

(ii) It is given that S is a maximum. Use differentiation to find $\dfrac{h}{x}$ in terms of R and L. [5]

6 A curve C has equation $y = \dfrac{\mathrm{e}^{kx}}{\mathrm{e}^{kx} - \mathrm{e}^{-kx}}$, where $x \neq 0$ and $k \in \mathbb{R}, k \neq 0$.

(i) Show that C has no stationary points. [2]

(ii) Find the equation of the tangent to C at the point where $y = 2$. [4]

(iii) Given that $k = 1$, find the acute angle between the tangent in **(ii)** and the y-axis. [2]

7 The functions f and g are defined by

$$\mathrm{f}: x \mapsto x^2 + 3x + 1,\ x \in \mathbb{R},\ x \leq -\frac{3}{2},$$

$$\mathrm{g}: x \mapsto 3 + \mathrm{e}^{-x},\ x \in \mathbb{R}.$$

(i) Show that f has an inverse. [1]

(ii) Give a reason why the composite function $\mathrm{f}^{-1}\mathrm{g}$ exists. Find $\mathrm{f}^{-1}\mathrm{g}(x)$ and state the domain of $\mathrm{f}^{-1}\mathrm{g}$. [6]

The function h is defined as follows.

$$\text{h}: x \mapsto \ln(x-k+1),\ x \in \mathbb{R},\ x \geq k, \quad \text{where } k < 0.$$

(iii) Find the value of k such that the range of hg is given by $(\ln 5, \infty)$. [2]

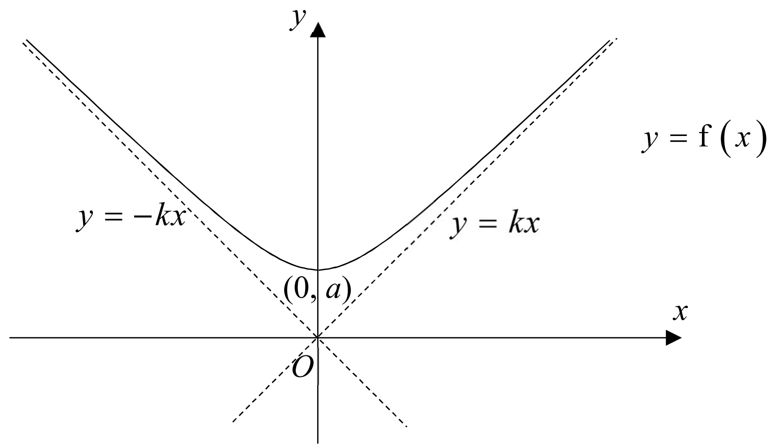
8 **(a)** It is given that $f(x) = e^{\tan^{-1}\frac{x}{2}}$.

(i) Show that $\left(4+x^2\right)f'(x) = 2f(x)$. [1]

(ii) By using repeated differentiation, find the Maclaurin series for $f(x)$, up to and including the term in x^2. Use this series to find an estimate for $e^{\frac{\pi}{6}}$ and show that the error in the approximation is about 3.3%. [6]

(b) It is given that $\mathrm{g}(x)=\dfrac{1}{\sqrt{2}\cos\left(\dfrac{x}{a}+\dfrac{\pi}{4}\right)}$, where a is a constant such that $a>1$. If x is sufficiently small, show that $\mathrm{g}(x)\approx 1+px+qx^2$ for constants p and q to be determined in terms of a. [4]

9 The diagram shows the graph of $y = \mathrm{f}(x)$. The curve has a turning point at $(0, a)$ and the lines $y = \pm kx$ are asymptotes to the curve, where a and k are positive real constants.



(i) On separate diagrams, sketch the graphs of $y = \frac{1}{k}\mathrm{f}(x + 2a)$ and $y = \mathrm{f}'(x)$, giving the equations of any asymptotes and the coordinates of any turning points and of any points of intersection with the axes. You should label the graphs clearly. [4]

[Question 9 continues on the next page.]

9 **[Continued]**

It is now given that $\mathrm{f}(x) = a\sqrt{1+x^2}$ and this holds for the rest of the question.

(ii) The region R is bounded by the curve $y = \mathrm{f}(x)$, the line $x = \sqrt{3}$ and the axes. A sculpture is made in the shape of the solid of revolution formed by rotating R through 2π radians about the y-axis. Find the volume of the sculpture in terms of a and π. [3]

(iii) Another region S is bounded by the curve $y = \dfrac{a}{\mathrm{f}\left(\dfrac{x}{3}\right)}$, the lines $x = \pm\, b\ (b > 0)$ and the x-axis. A second sculpture takes the shape of the solid of revolution formed by rotating S through 2π radians about the x-axis. Given that the volume of the first sculpture found in part **(ii)** is ten times as great as the volume of the second sculpture, find an expression for b in terms of a. [4]

10 Wendie attends training sessions in preparation for a running race. In the first session, she runs a distance of 1000 metres in exactly 5 minutes. For each subsequent session, the distance run is $x\%$ more than the distance run in the previous session and the time taken is 1.5 minutes longer than the time taken for the previous session.

(i) It is given that $x = 8$.

(a) Find the distance she runs in the 15th session, giving your answer correct to 2 decimal places. [1]

(b) Find the minimum number of training sessions that she needs to attend in order to accumulate a total running time of 15 hours at the end of these sessions. [3]

(c) Find, in terms of *n,* the time taken to complete the distance in the *n*th session. Hence find the value of *n* such that Wendie's average speed of running in the *n*th session first exceeds 220 metres per minute. [3]

(ii) Find the value of *x* if Wendie has run a total of 250 kilometres at the end of 25 training sessions. Give your answer correct to 2 decimal places. [3]

[Question 10 continues on the next page.]

10 [Continued]

(iii) Wendie modifies her training plan such that she runs a distance of 2000 metres in her first training session, and decreases the distance she runs in each subsequent session by 10% of the distance run in the previous session. Show that she will never accumulate a total distance exceeding 20 kilometres. [2]

11 In a chemical reaction, a compound X is formed from two compounds Y and Z.

The masses in grams of X, Y and Z present at time t seconds after the start of the reaction are x, $15-x$ and $25-x$ respectively. At any time, the rate of formation of X is proportional to the product of masses of Y and Z present at the time. It is given that $x=0$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=3$ when $t=0$.

(i) Write down a differential equation relating x and t and find the value of the constant of proportionality. [2]

[Question 11 continues on the next page.]

11 [Continued]

(ii) Solve the differential equation found in part **(i)** and obtain an expression for x in terms of t. [7]

(iii) Sketch the graph of x against t. [1]

(iv) Find the time taken for the mass in grams of X to be equal to the mass in grams of Y. [2]

(v) What happens to the mass of X for large values of t? [1]

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